Applications of Symmetry to Robotics

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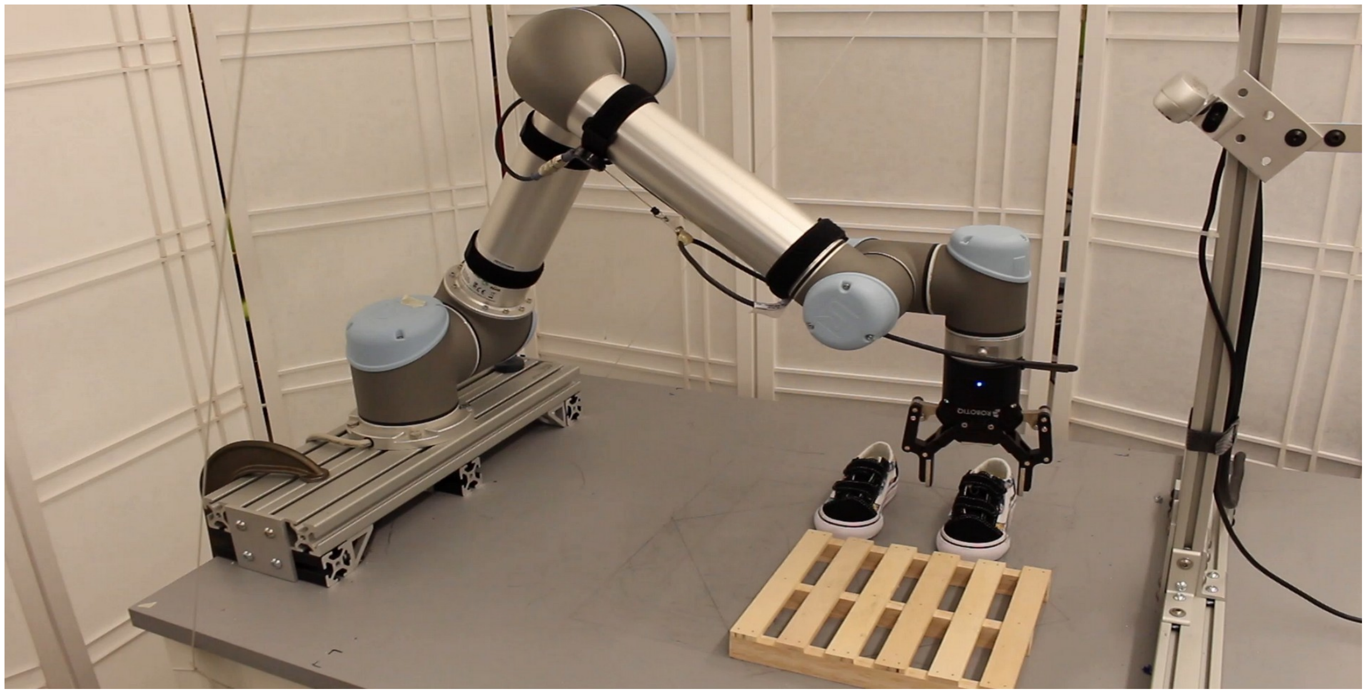
Problem: Find Robot Control Policy

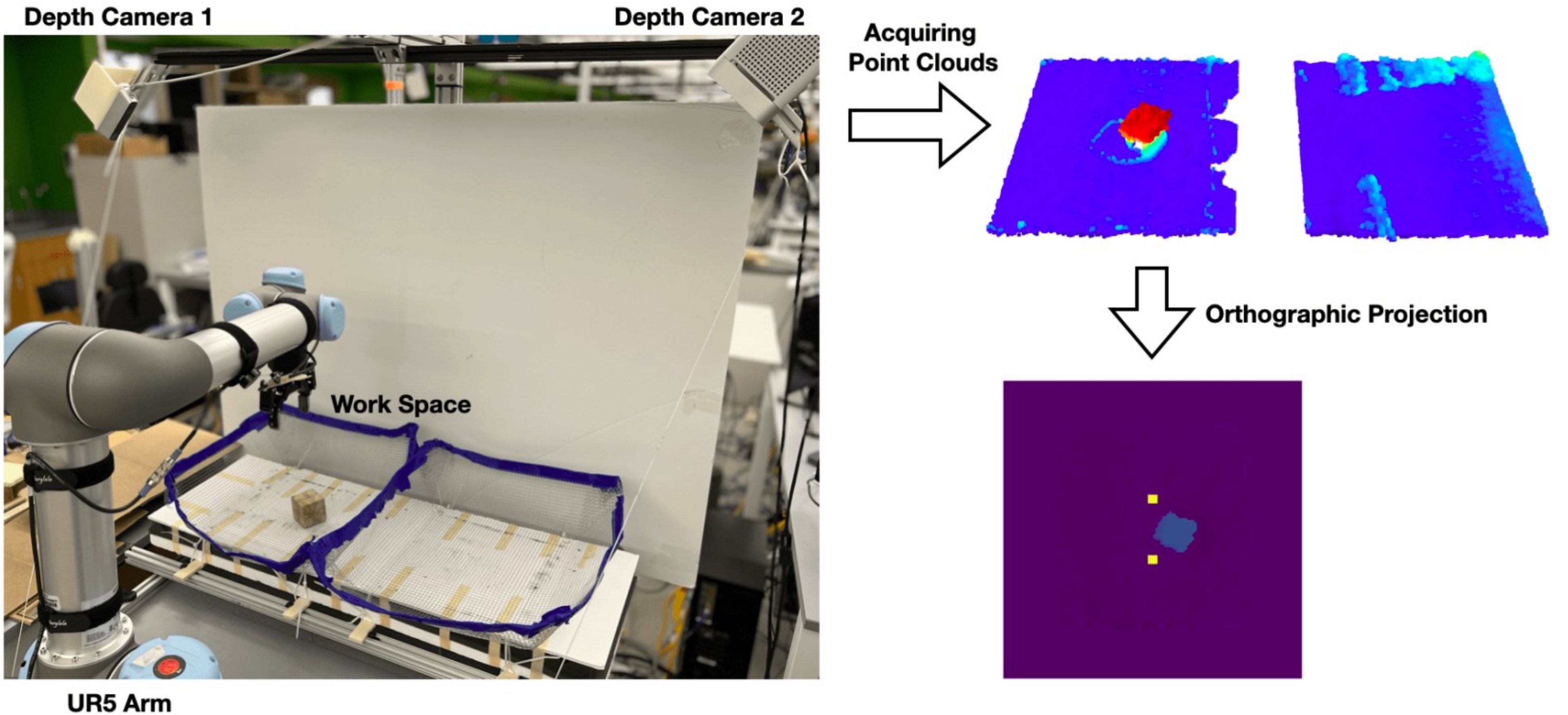


Learn Control Policy: $a = \pi(s)$

Control signal

Image, lidar,
force, tactile, etc.

Robotics Problems Often Have Geometric Structure



Symmetry In Transition Dynamics

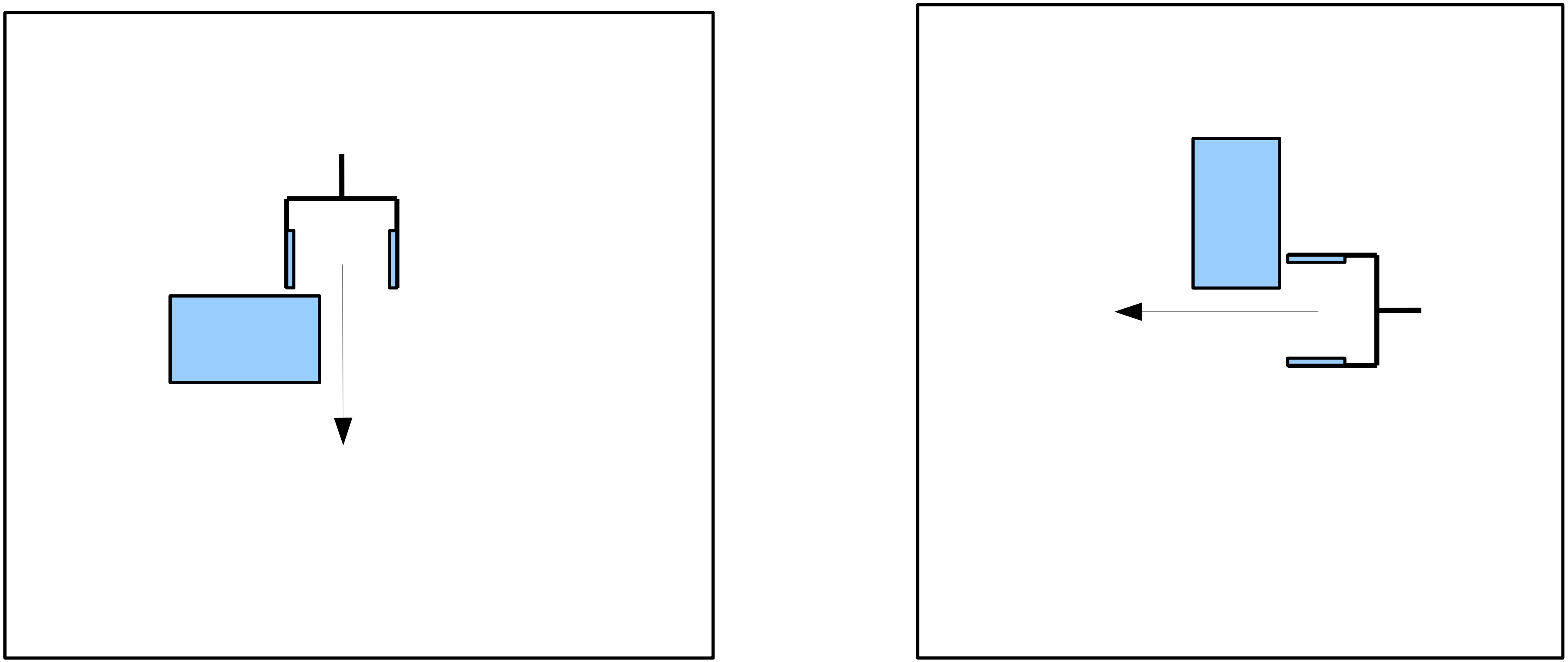


Image: $s \in \mathbb{R}^{h \times w}$

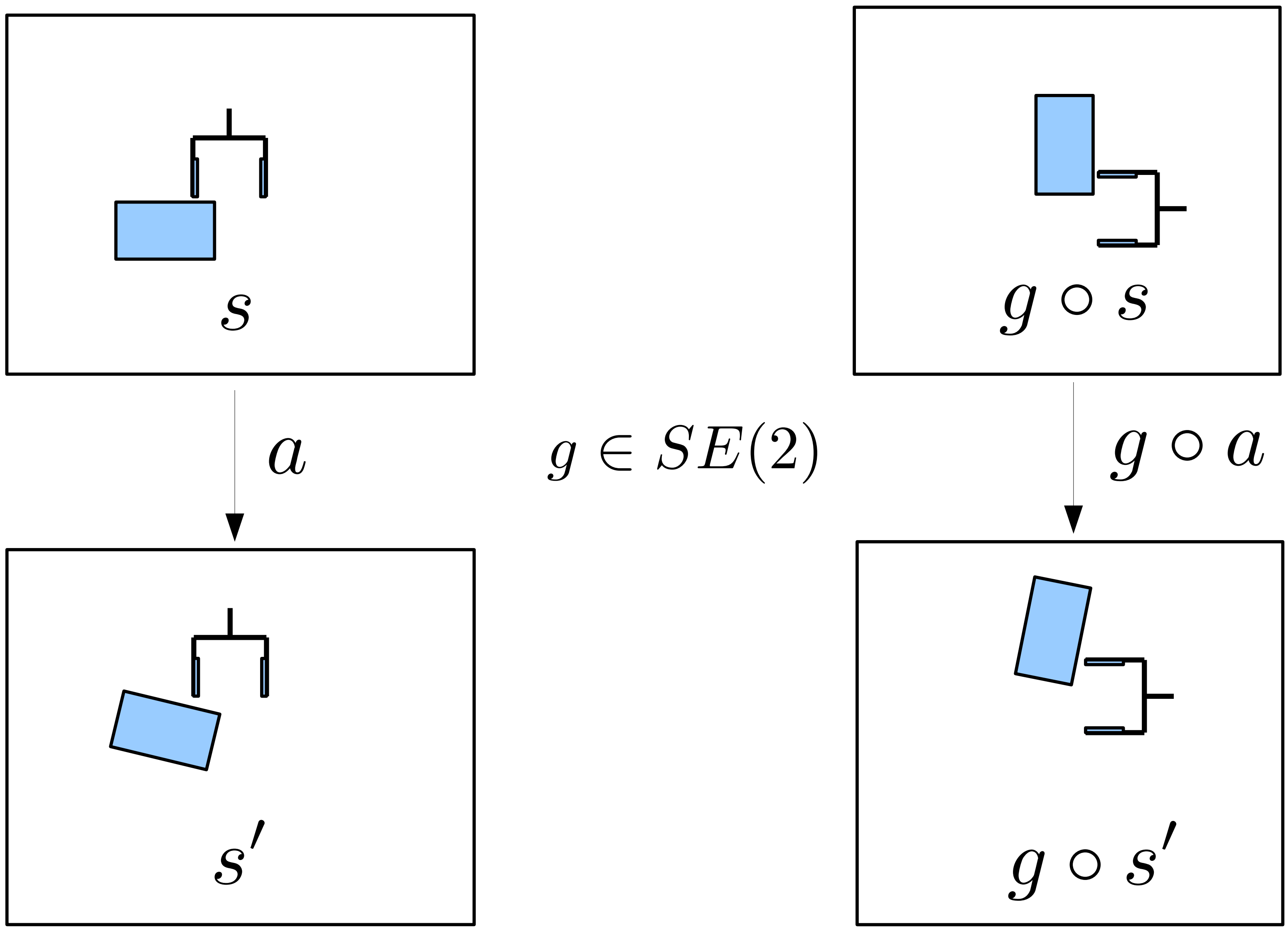
Velocity: $a \in \mathbb{R}^2$

$g \in SE(2)$

$g \circ s$

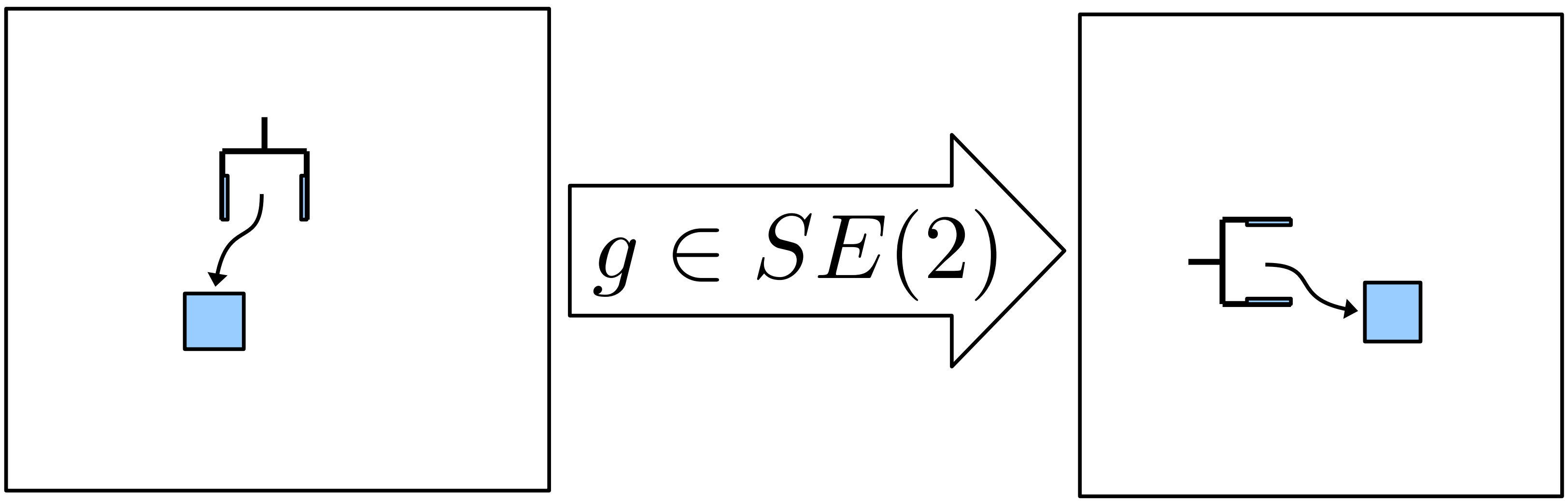
$g \circ a$

Symmetry In Transition Dynamics



$$p(s'|s,a) = p(g \circ s'|g \circ s, g \circ a)$$

Translates to Policy Symmetries



Policy: $a = \pi(s)$

Policy Symmetry: $g \circ \pi(s) = \pi(g \circ s)$

Symmetric MDPs Have Symmetric Optimal Vaue Functions and Policies

Definition 4.1 (G-invariant MDP). *A G-invariant MDP $\mathcal{M}_G = (S, A, T, R, G)$ is an MDP $\mathcal{M} = (S, A, T, R)$ that satisfies the following conditions:*

1. Reward Invariance: *The reward function is invariant to the action of the group element $g \in G$, $R(s, a) = R(gs, ga)$.*

2. Transition Invariance: *The transition function is invariant to the action of the group element $g \in G$, $T(s, a, s') = T(gs, ga, gs')$.*

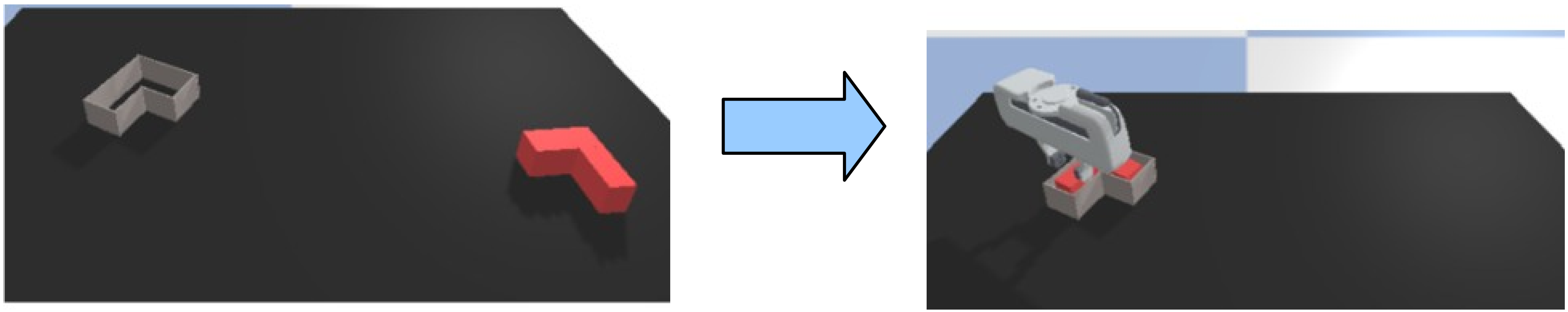
Proposition 4.1. *Let $\mathcal{M}_G$ be a group-invariant MDP. Then its optimal Q-function is group invariant, $Q^*(s, a) = Q^*(gs, ga)$, and its optimal policy is group-equivariant, $\pi^*(gs) = g\pi^*(s)$, for any $g \in G$.*

Hard code symmetries into solutions: $Q(s, a) = Q(g \circ s, g \circ a)$

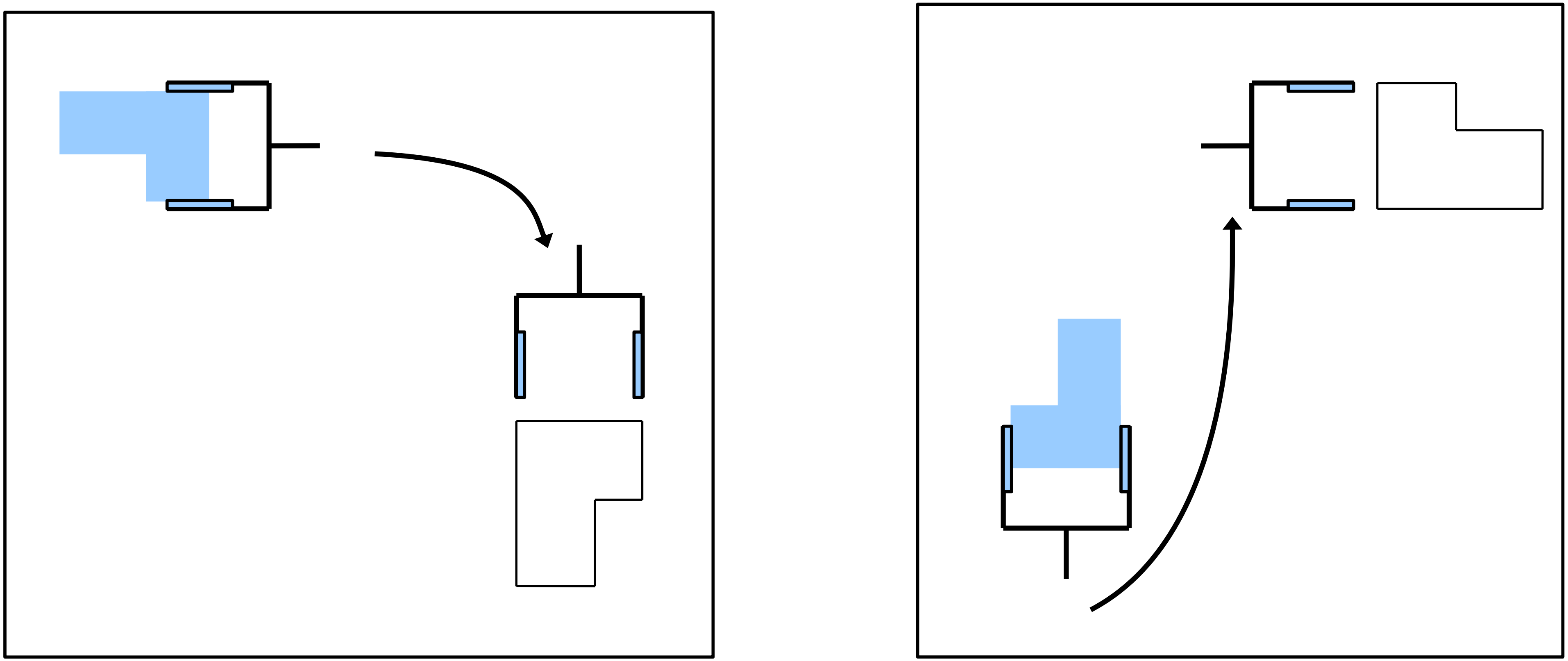
$$g \circ \pi(s) = \pi(g \circ s)$$

Wang, Walters, Platt, *SO(2)-Equivariant Reinforcement Learning*, ICLR 2022

Object Factored Symmetries



Object Factored Symmetries



$$g \in SE(2) \times SE(2)$$

Symmetries in SE(3)

Gameplan: Use escnn, e3nn, etc.

General E(2)-Equivariant Steerable CNNs

Documentation | Experiments | Paper | Thesis | new escnn library

E(n)-equivariant Steerable CNNs (*escnn*)

Documentation | Paper ICLR 22 | Paper NeurIPS 19 | e2cnn library | e2cnn experiments | Thesis

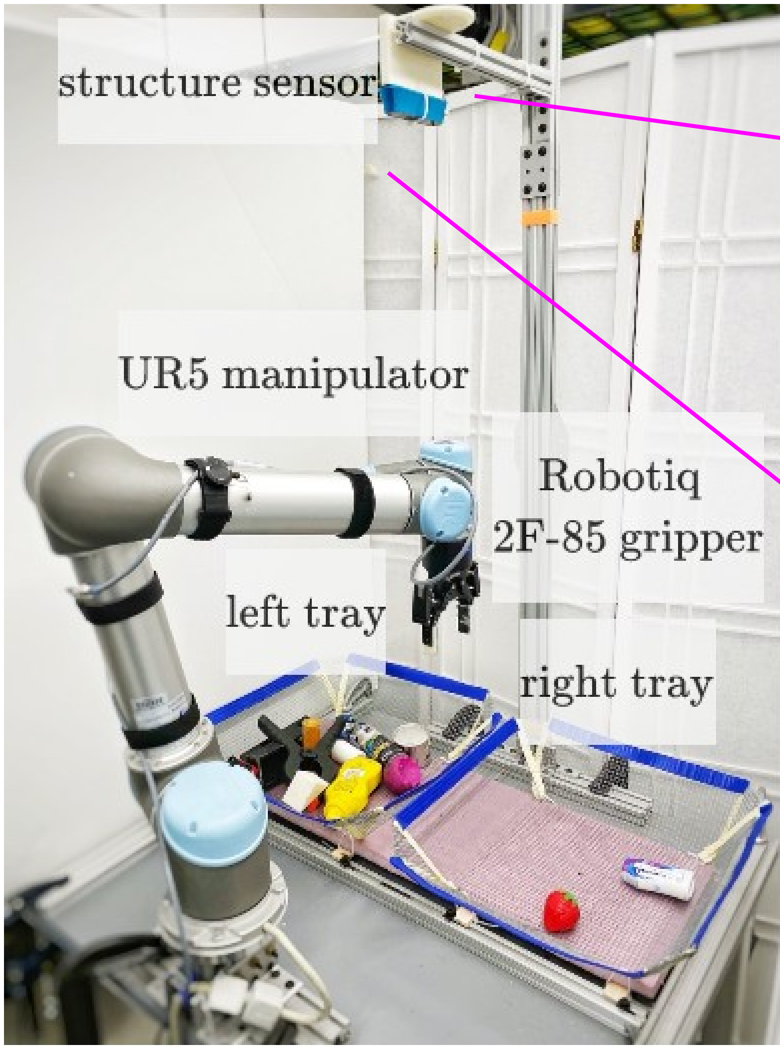
Euclidean neural networks

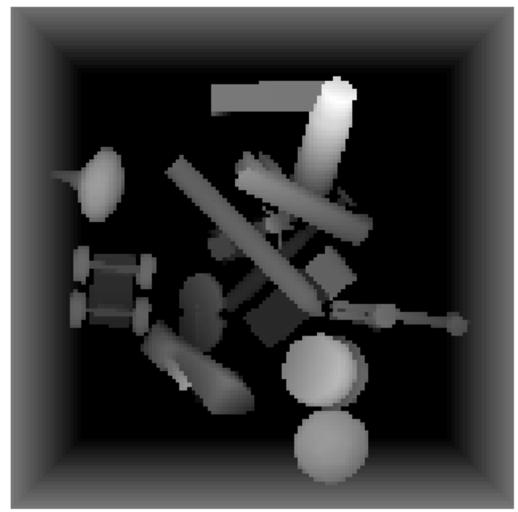
coverage 96% DOI 10.5281/zenodo.7430260

Documentation | Code | ChangeLog | Colab

The aim of this library is to help the development of E(3) equivariant neural networks. It contains fundamental mathematical operations such as tensor products and spherical harmonics.

#1) Grasp Learning in SE(2)



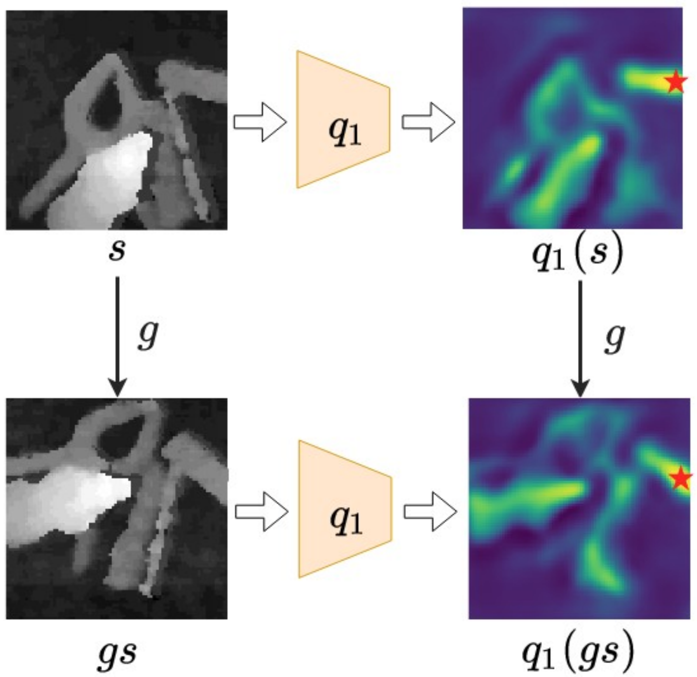


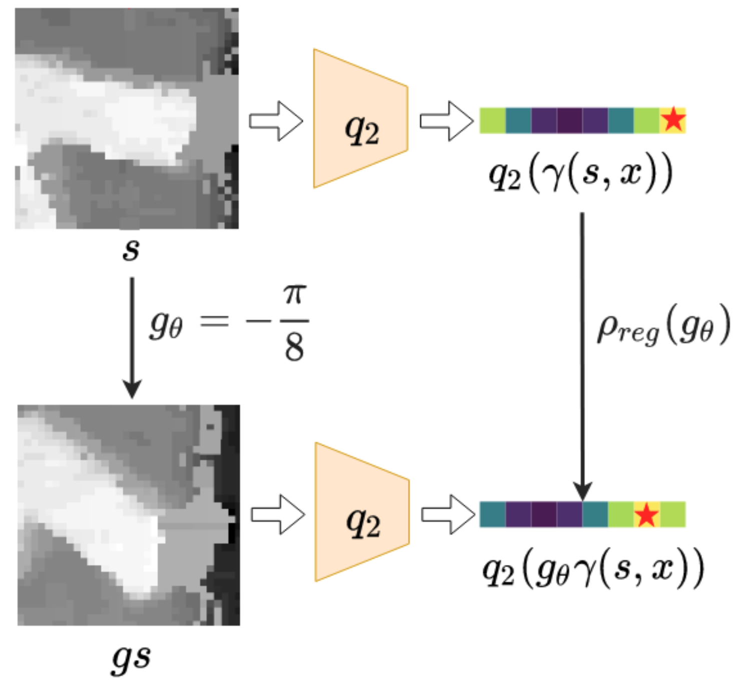
Depth image

Grasp pose
(x, y, θ)

Zhu, Wang, Biza, Su, Walters, Platt. *Sample Efficient Grasp Learning Using Equivariant Models*. RSS 2022

#1) Grasp Learning in SE(2)



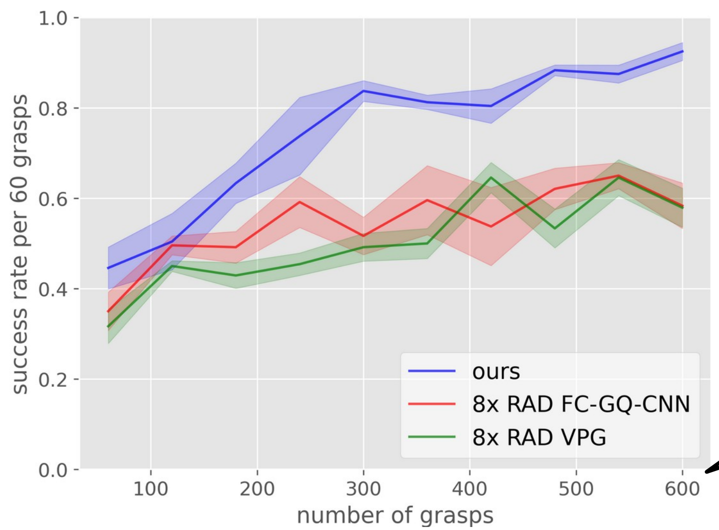


Type of equivalence: as input image rotates, output image also rotates.

Type of equivalence: as input image rotates, output vector does circular shift.

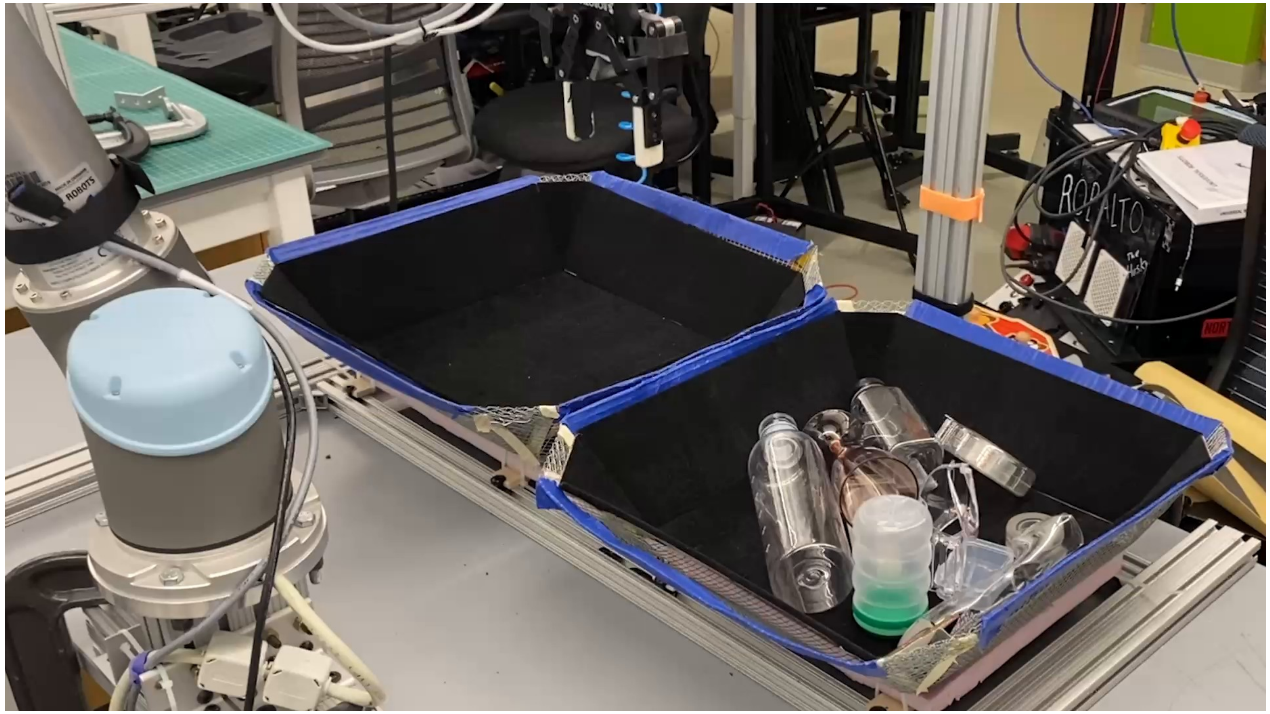
Zhu, Wang, Biza, Su, Walters, Platt. *Sample Efficient Grasp Learning Using Equivariant Models*. RSS 2022

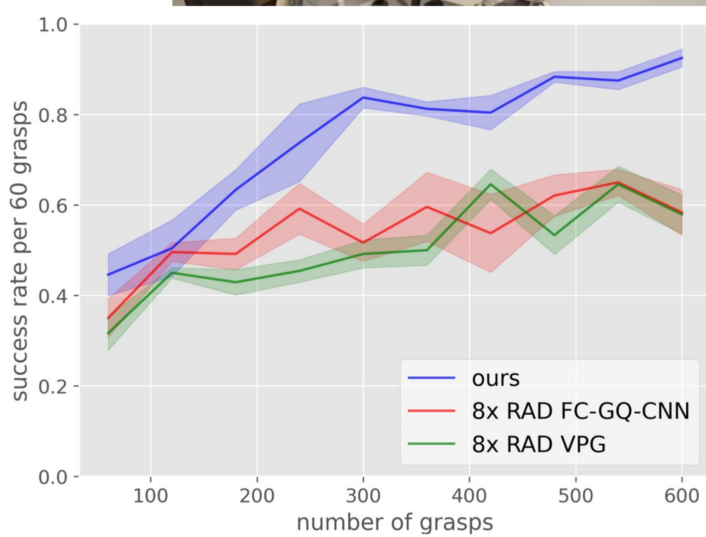
#1) Grasp Learning in SE(2)



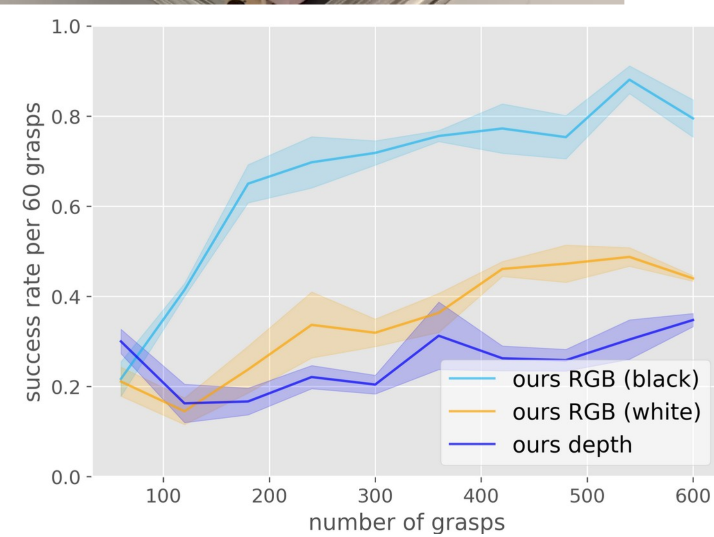
600 grasps = 1 hr, 20 min

#1) Grasp Learning in SE(2)



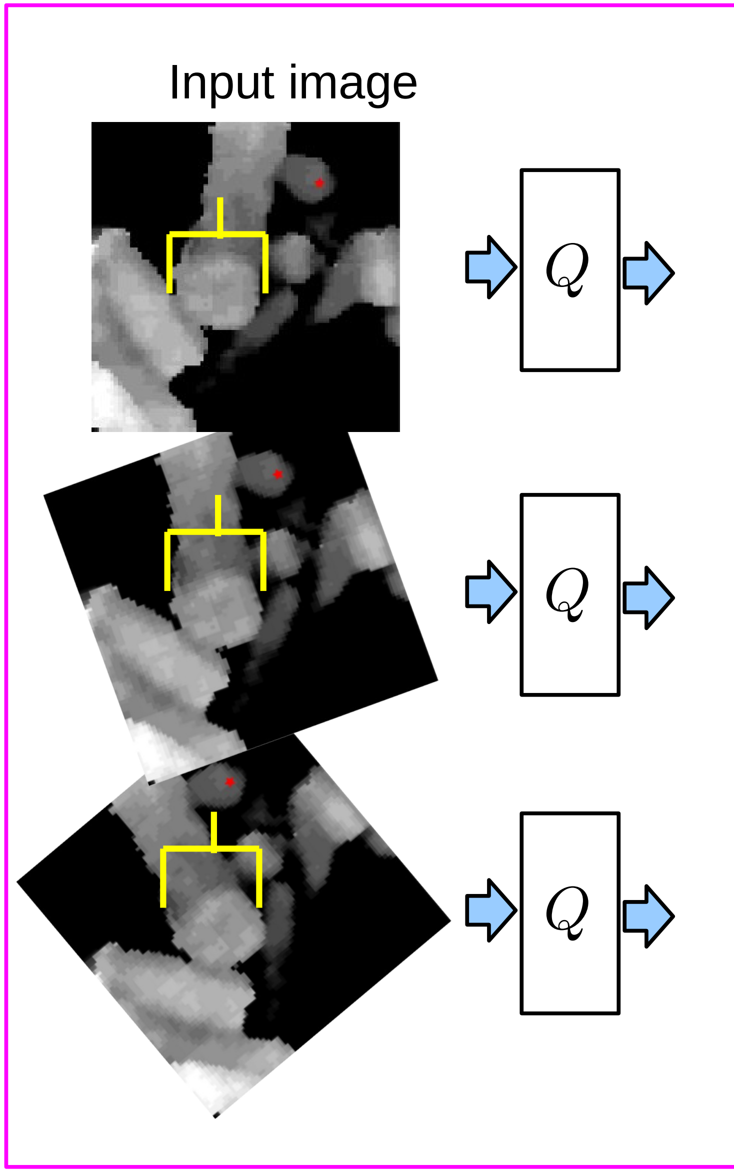


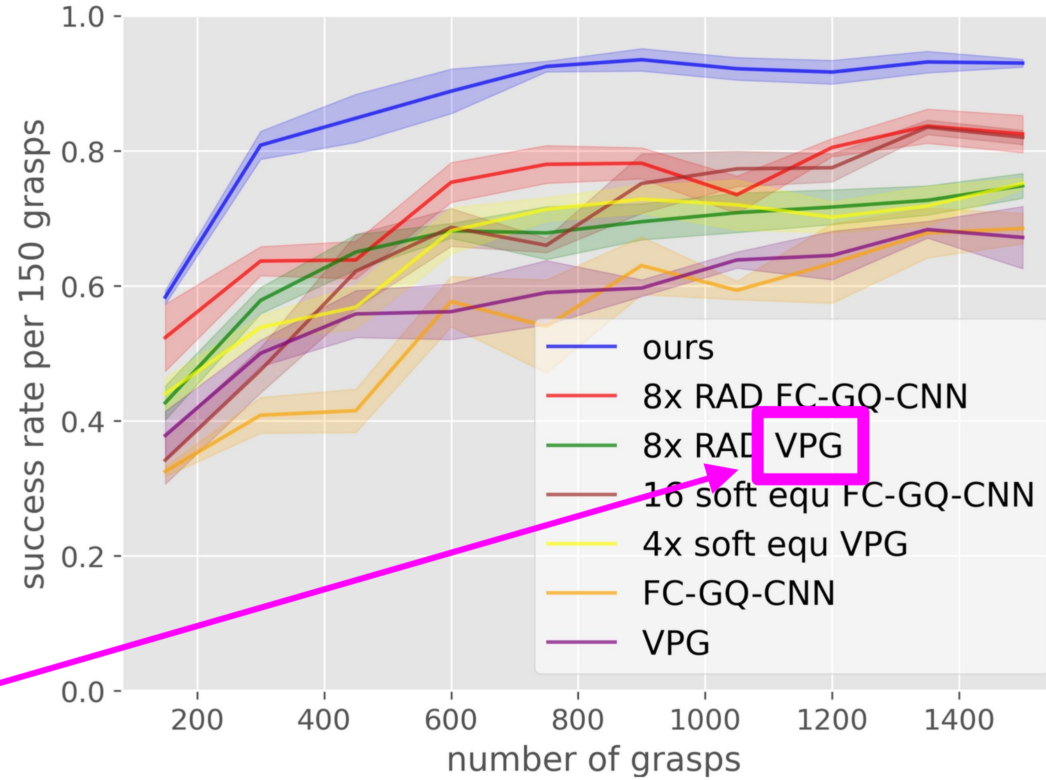
opaque



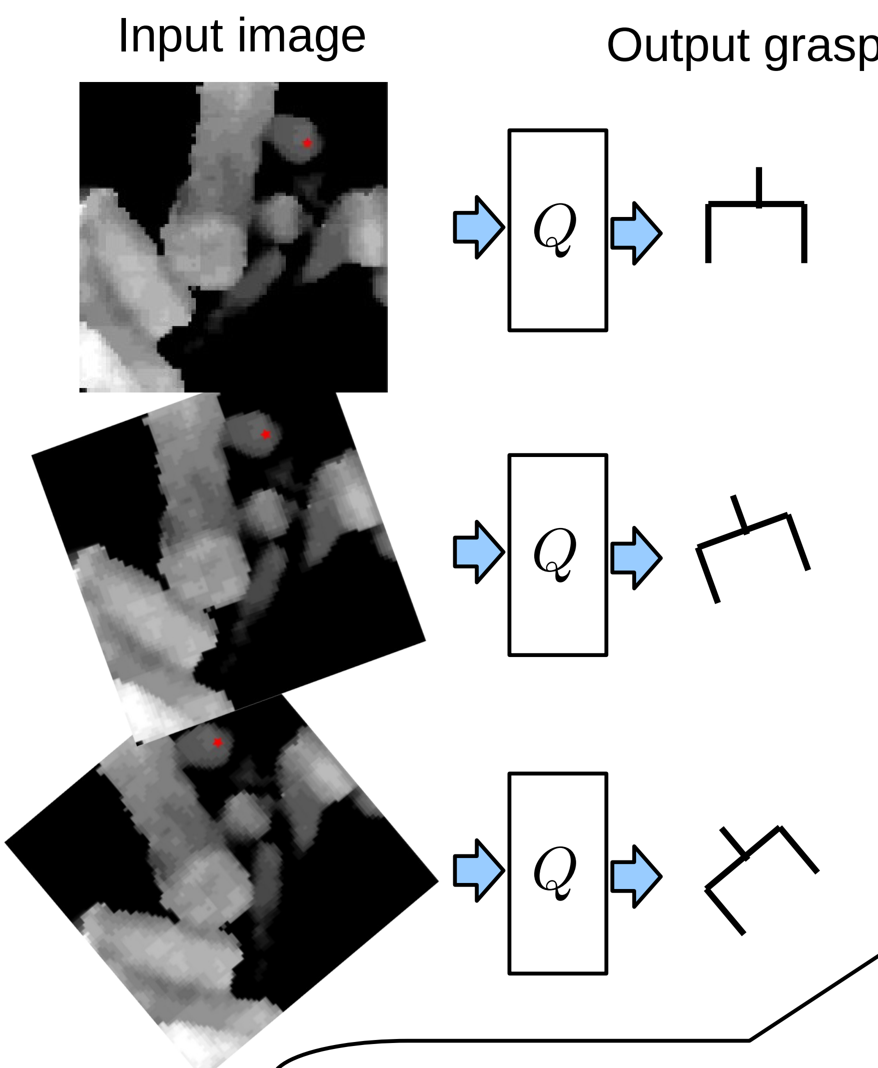
transparent

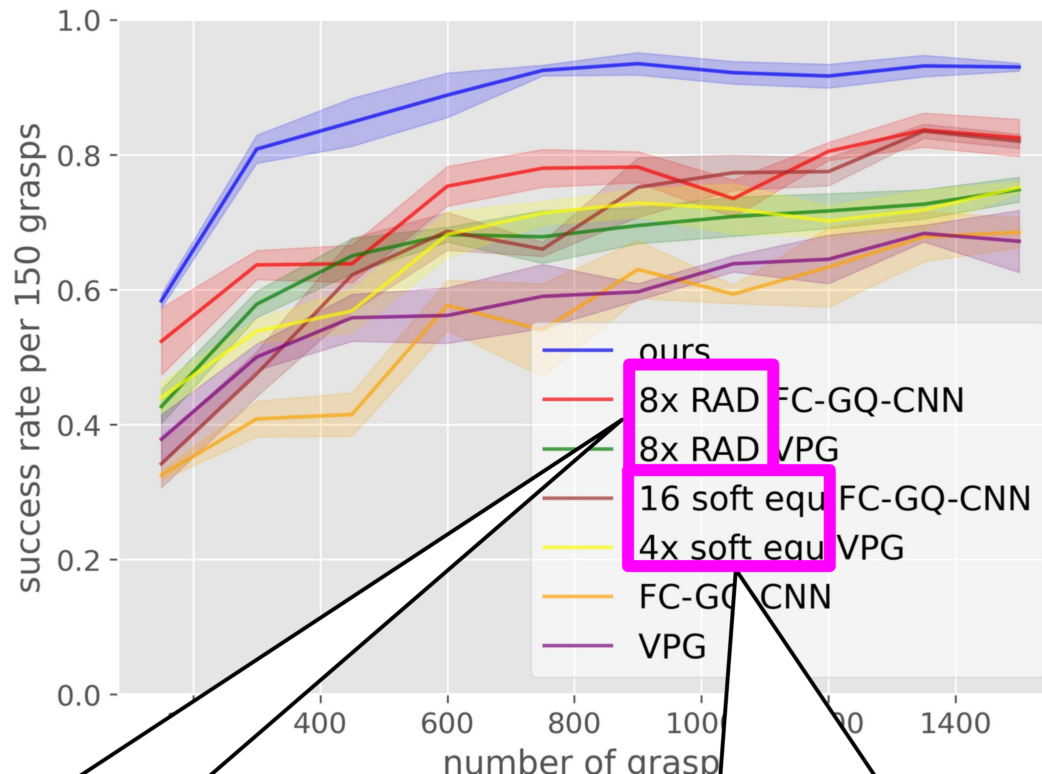
Equivariance via canonicalization





Data Augmentation

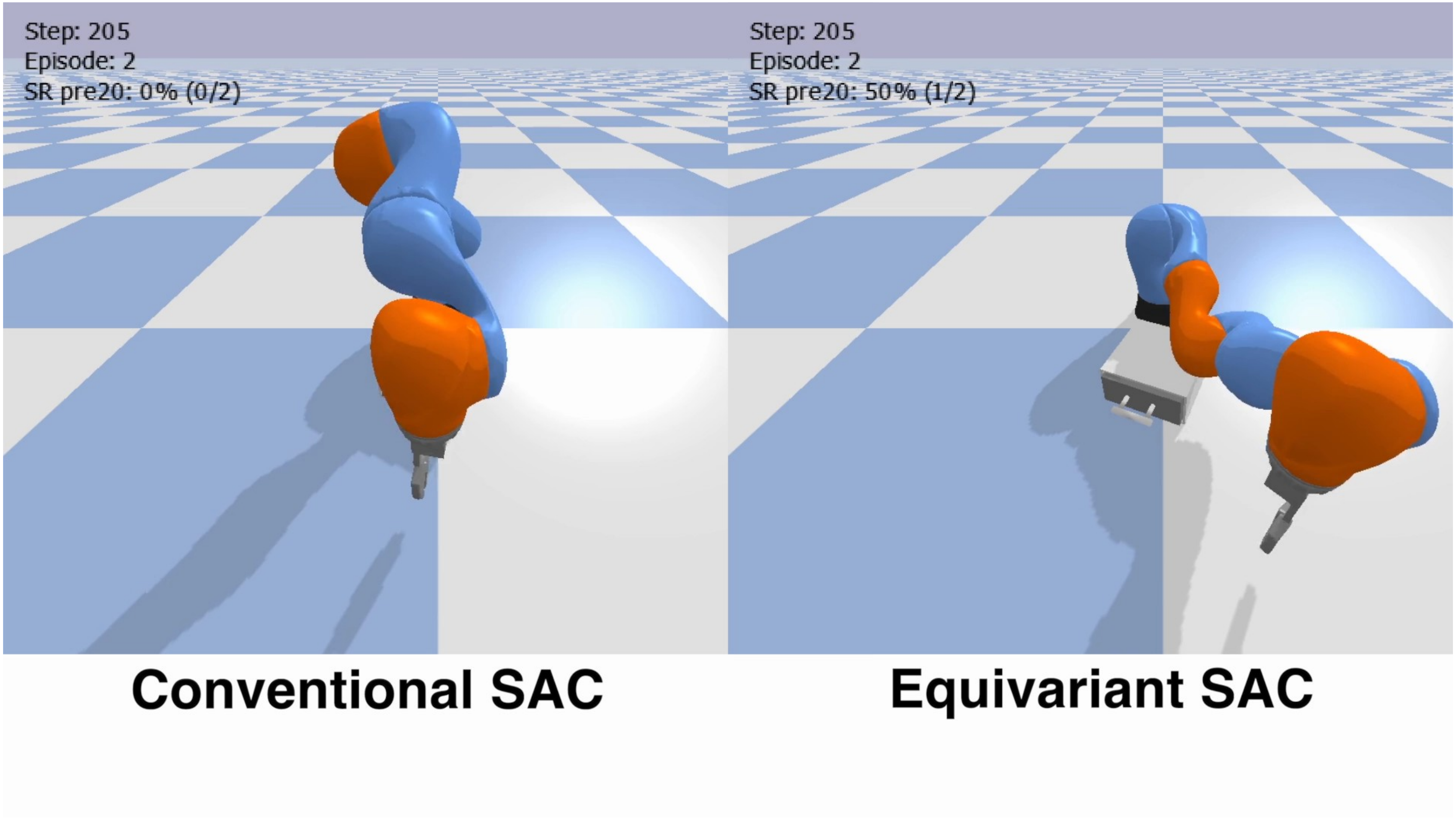




8 SGD steps per grasp
-- each step is performed on a minibatch
-- each frame in mb is translated/rotated

16 or 4 SGD steps per grasp
-- each step is performed on a 1/16 minibatch
-- each frame in mb is augmented 16 times

#2) O(2) Equivariant SAC



Wang, Walters, Platt, SO(2)-Equivariant Reinforcement Learning, ICLR 2022

Wang, Jia, Zhu, Walters, Platt, On-Robot Policy Learning with O(2)-Equivariant SAC, arXiv preprint arXiv:2203.04923

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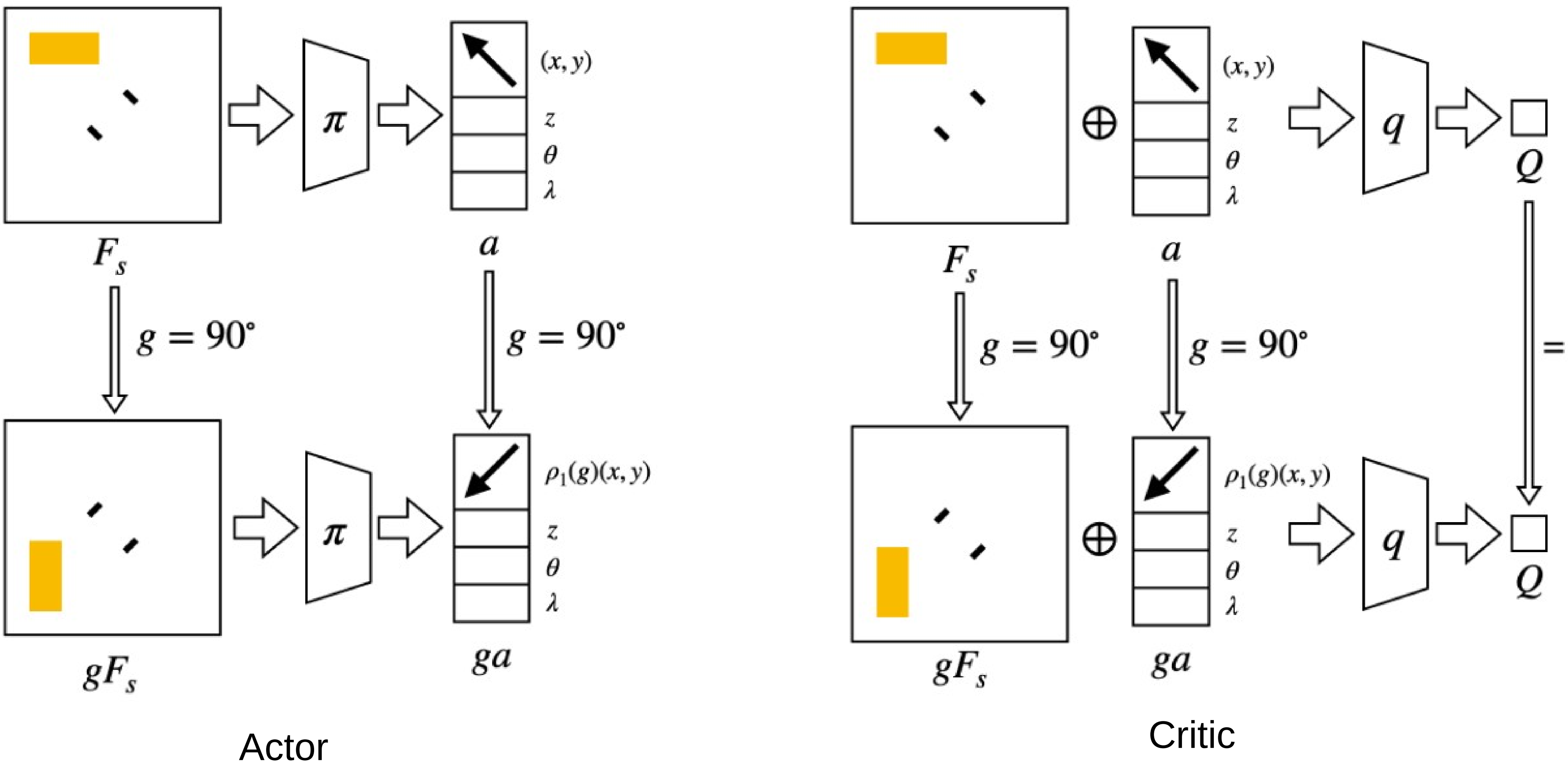

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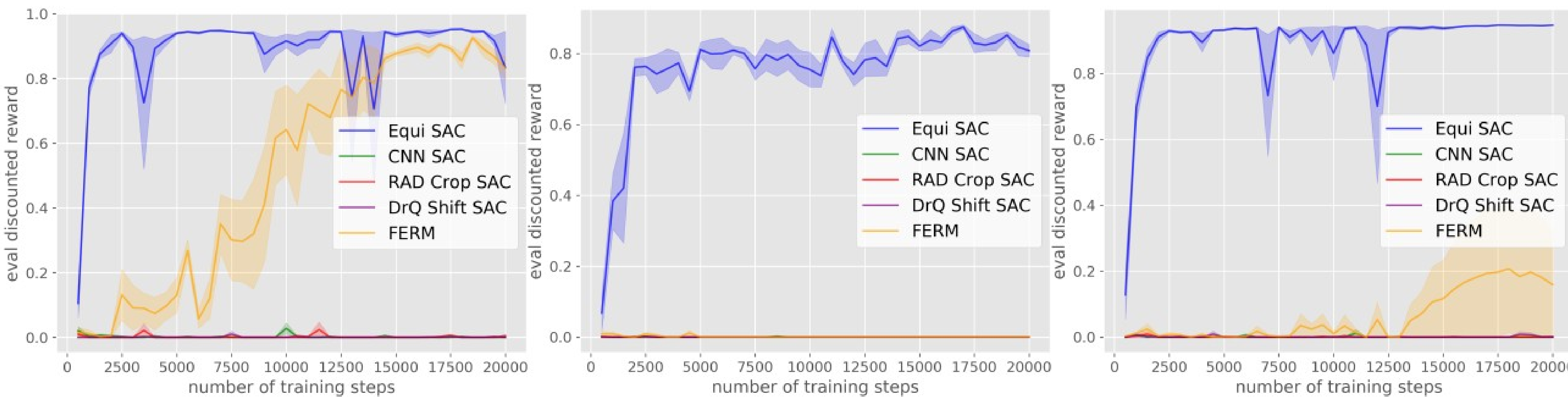
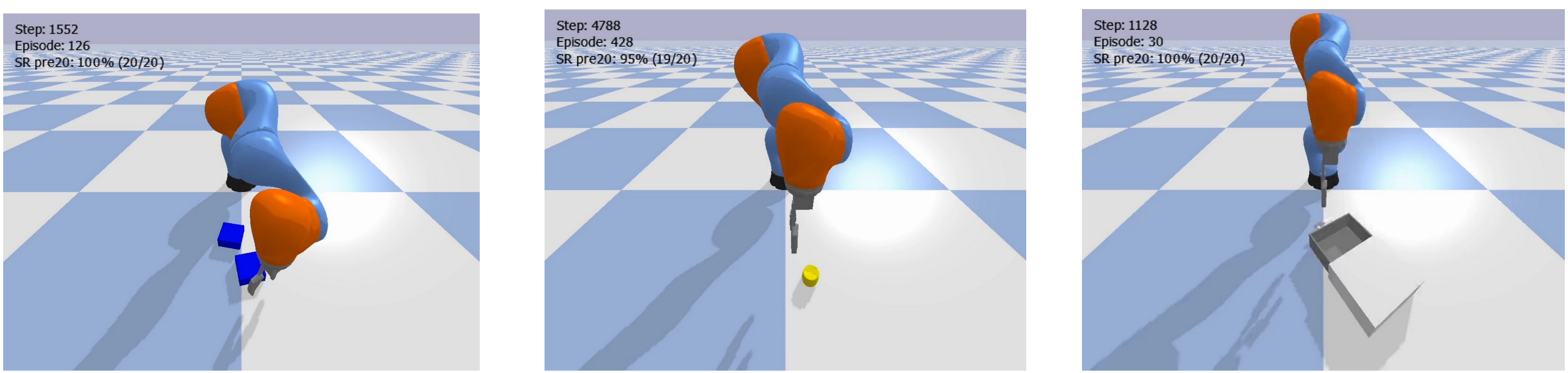


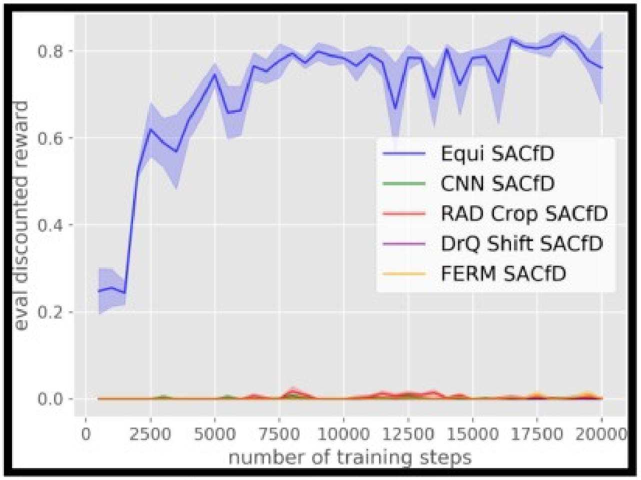
(a) Block Pulling (b) Object Picking (c) Drawer Opening



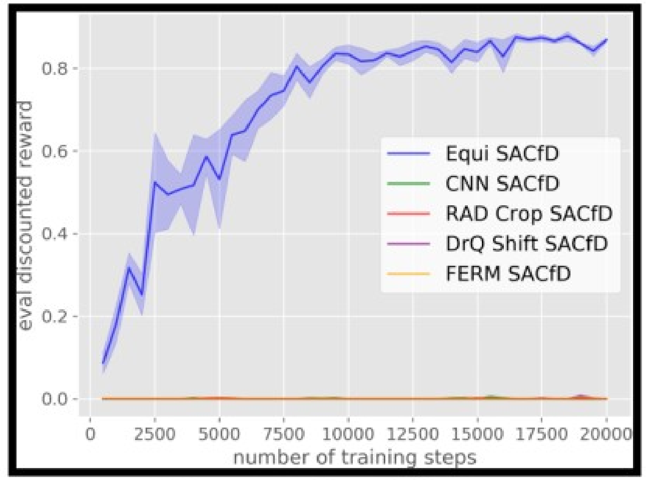
Wang, Walters, Platt, SO(2)-Equivariant Reinforcement Learning, ICLR 2022

Wang, Jia, Zhu, Walters, Platt, On-Robot Policy Learning with O(2)-Equivariant SAC, CoRL 2022

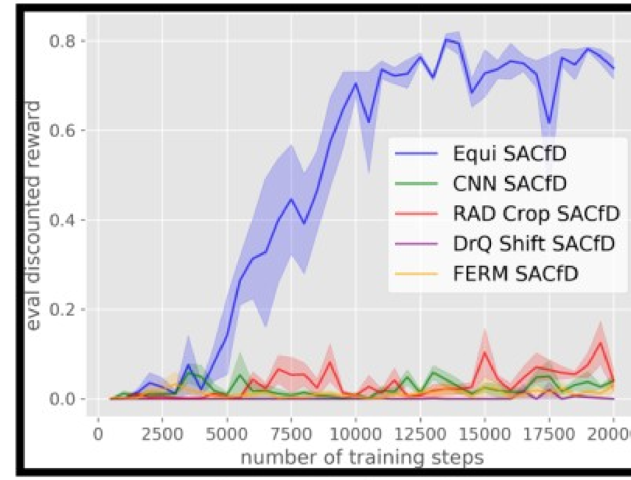
#2) O(2) Equivariant SAC



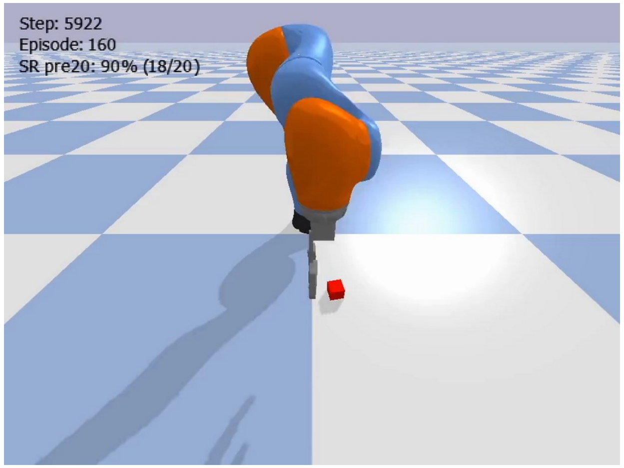
Block Stacking

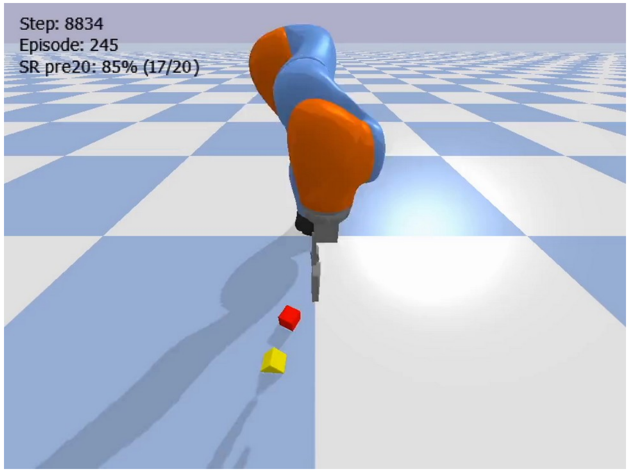


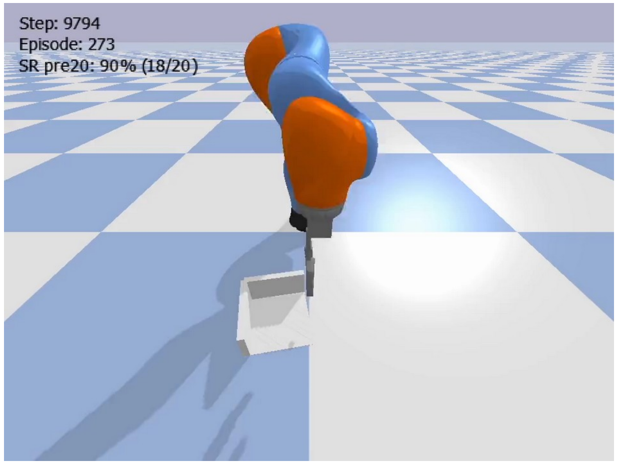
House Building



Corner Picking







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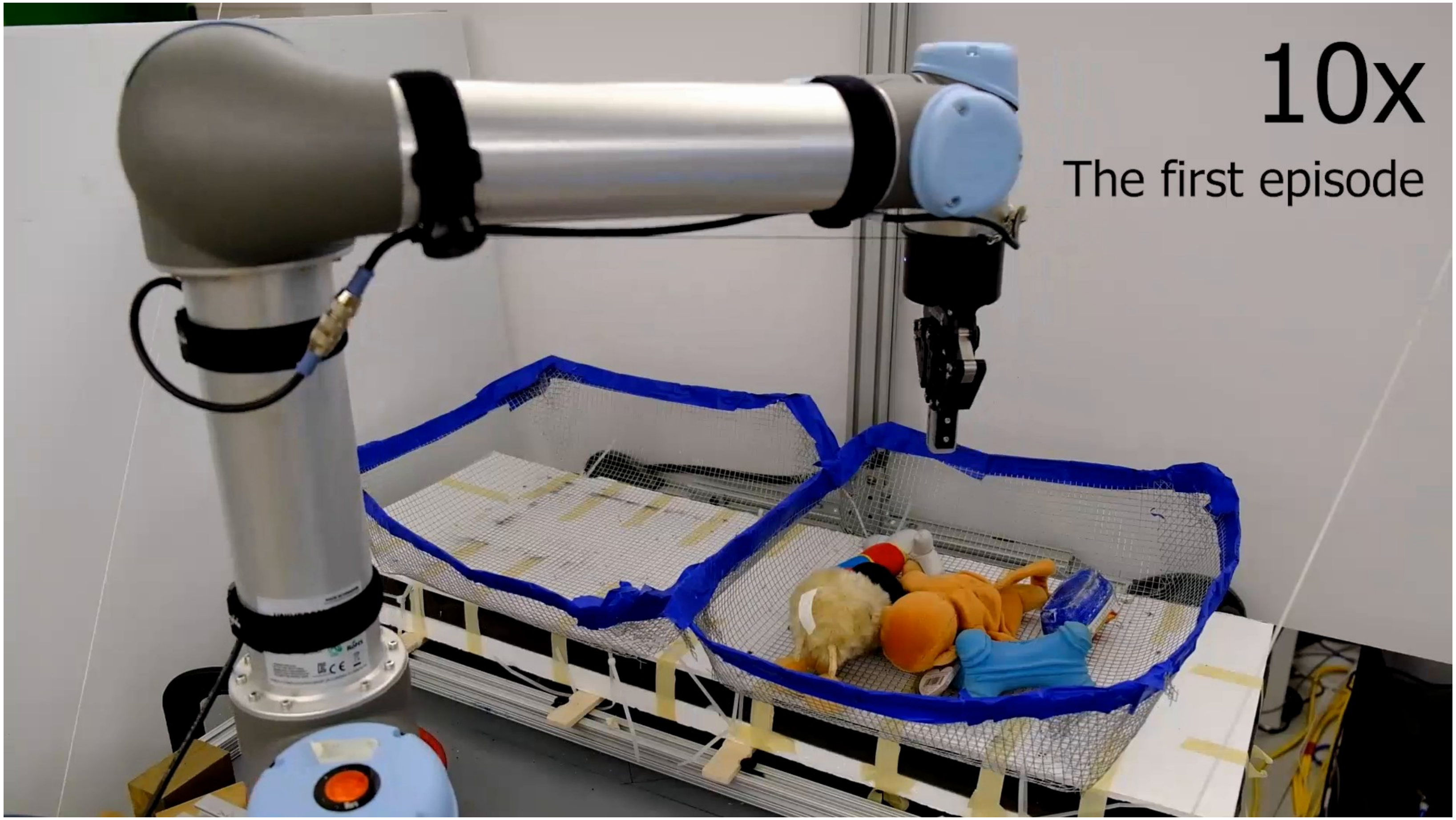
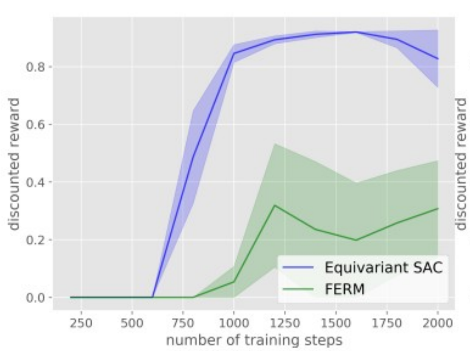
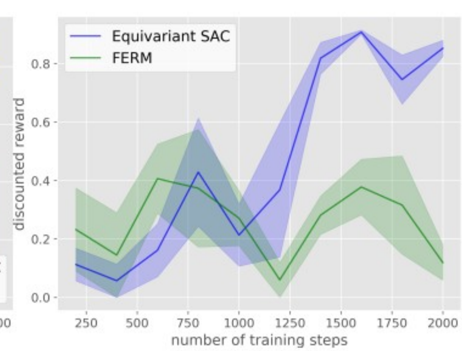
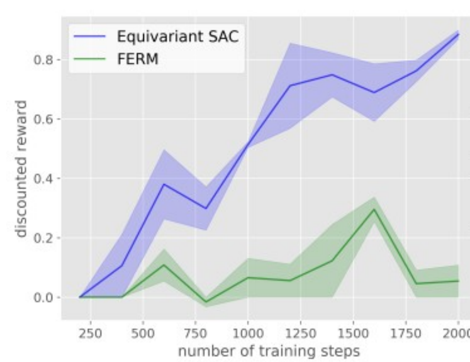




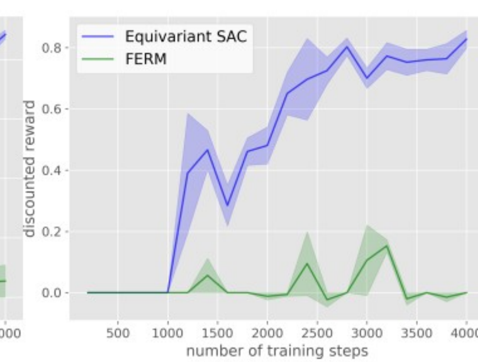
(a) Block Picking



(b) Clutter Grasping

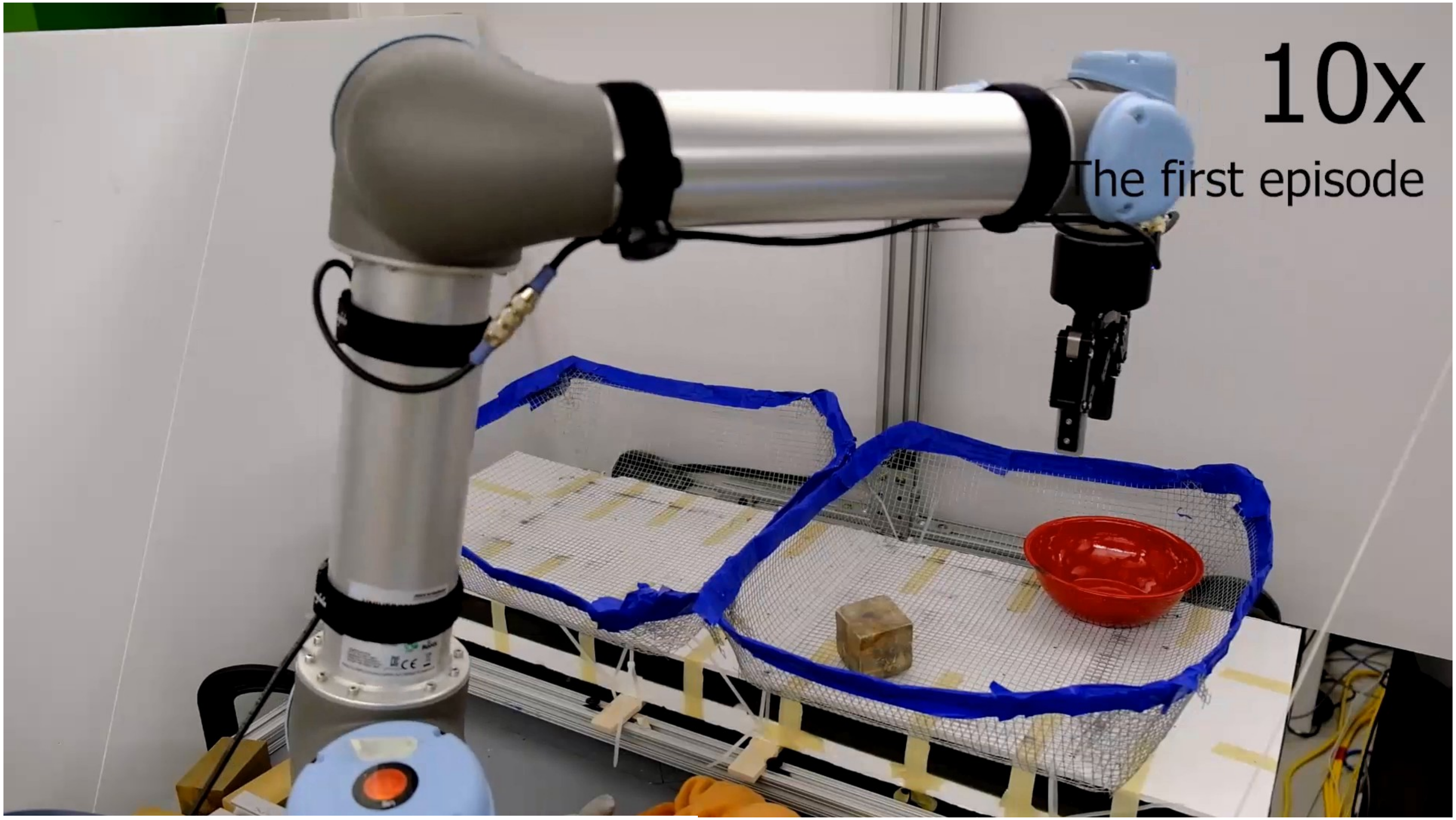


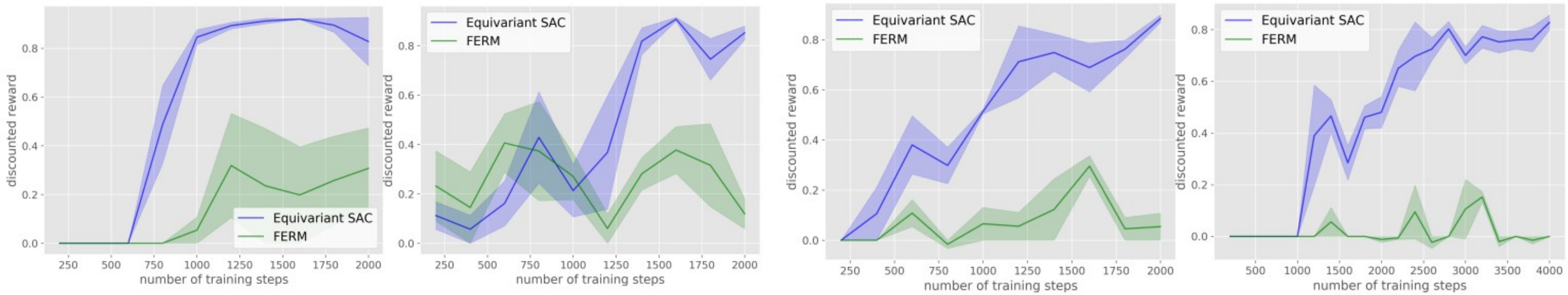
(c) Block Pushing



(d) Block in Bowl

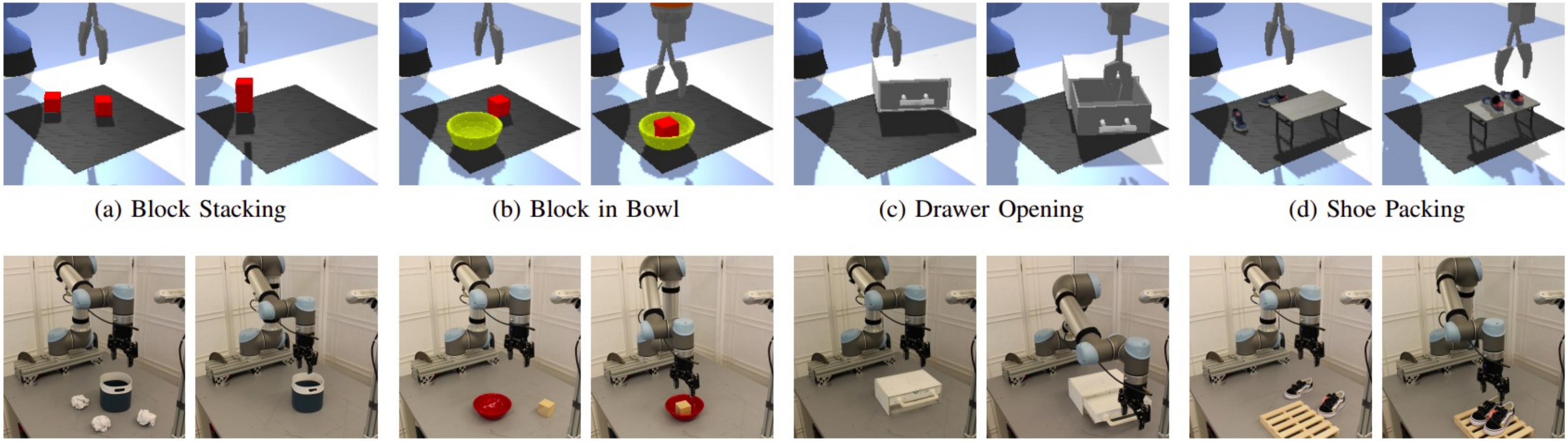
#2) O(2) Equivariant SAC





(a) Block Picking (b) Clutter Grasping (c) Block Pushing (d) Block in Bowl

#3) O(2) Equivariant IL



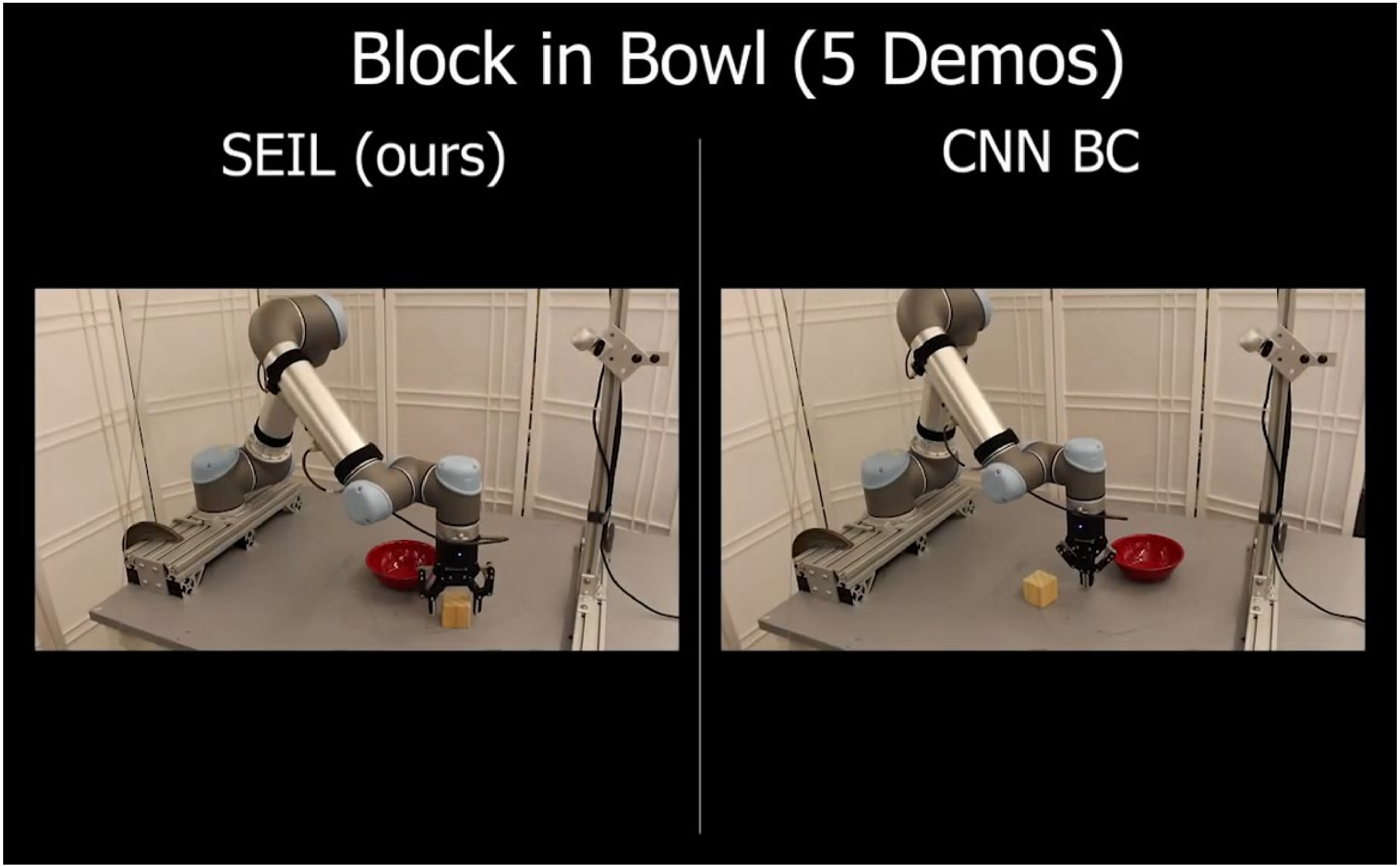
(a) Block Stacking (b) Block in Bowl (c) Drawer Opening (d) Shoe Packing

(e) Table Tidying (f) Block in Bowl (g) Drawer Opening (h) Shoe Packing

	Block Stacking				Block in Bowl				Drawer Opening				Shoe Packing			
Method	1	5	10	100	1	5	10	100	1	5	10	100	1	5	10	100
CNN BC	18.5	73.5	79.0	90.5	19.0	93.5	99.0	**100**	31.0	66.5	76.0	88.5	2.0	4.7	12.0	20.0
Implicit BC	11.0	9.5	51.0	80.5	13.0	**99.5**	**100**	**100**	31.0	63.5	71.5	81.5	0.5	5.5	12.0	13.0
CNN BC + TS	41.0	75.0	87.0	92.0	52.2	91.0	96.5	98.5	53.5	76.0	75.0	84.5	7.5	13.0	22.0	26.0
Equi BC (Ours)	33.5	87.5	93.0	**100**	46.5	**99.5**	99.5	**100**	62.5	88.5	91.0	**100**	1.5	22.5	39.5	**75.0**
SEIL (Ours)	**71.5**	**99.5**	**98.5**	**100**	**75.0**	98.0	**100**	**100**	**78.5**	87.5	**93.5**	96.5	**16.5**	**57.3**	**68.0**	72.0

Jia et al., SEIL: Simulation-augmented Equivariant Imitation Learning, ICRA 2023

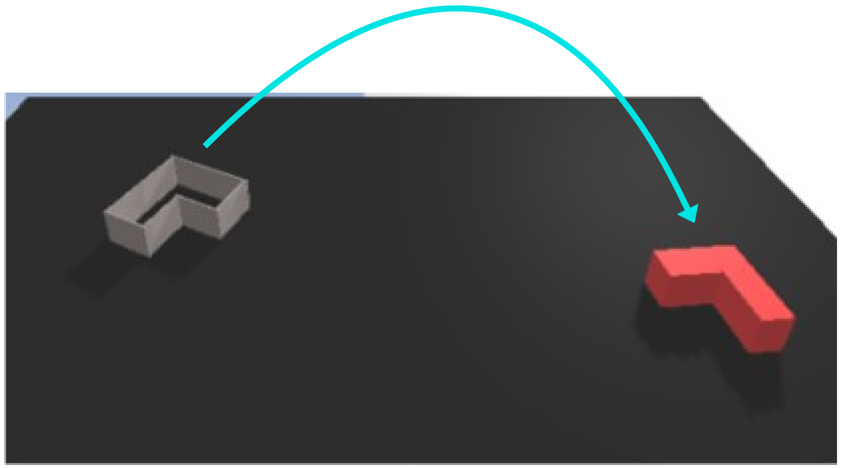
#3) O(2) Equivariant IL



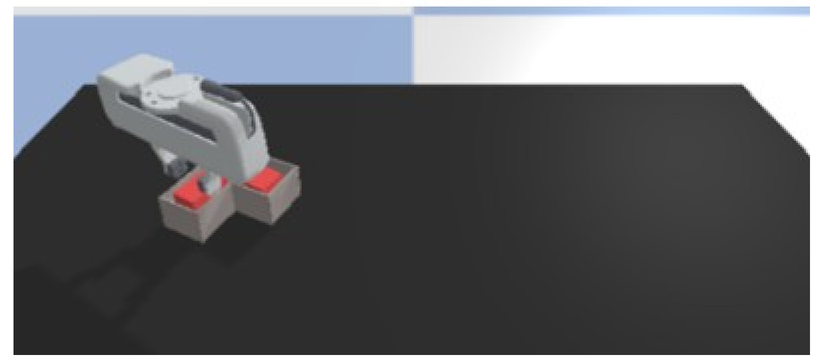
Jia et al., SEIL: Simulation-augmented Equivariant Imitation Learning, ICRA 2023

#4) Pick and Place

Demonstration

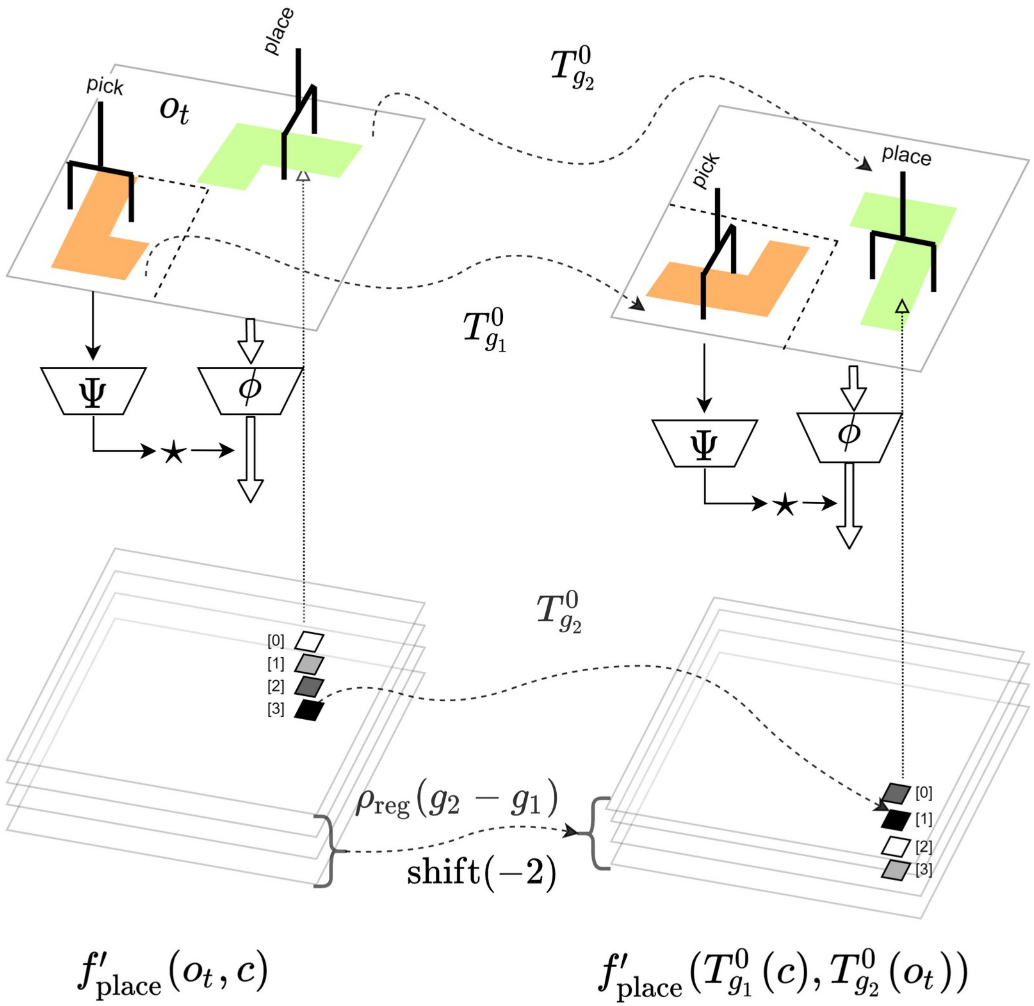


Control Policy



Huang, Wang, Walters, Platt, *Equivariant Transporter Network*, RSS 2022

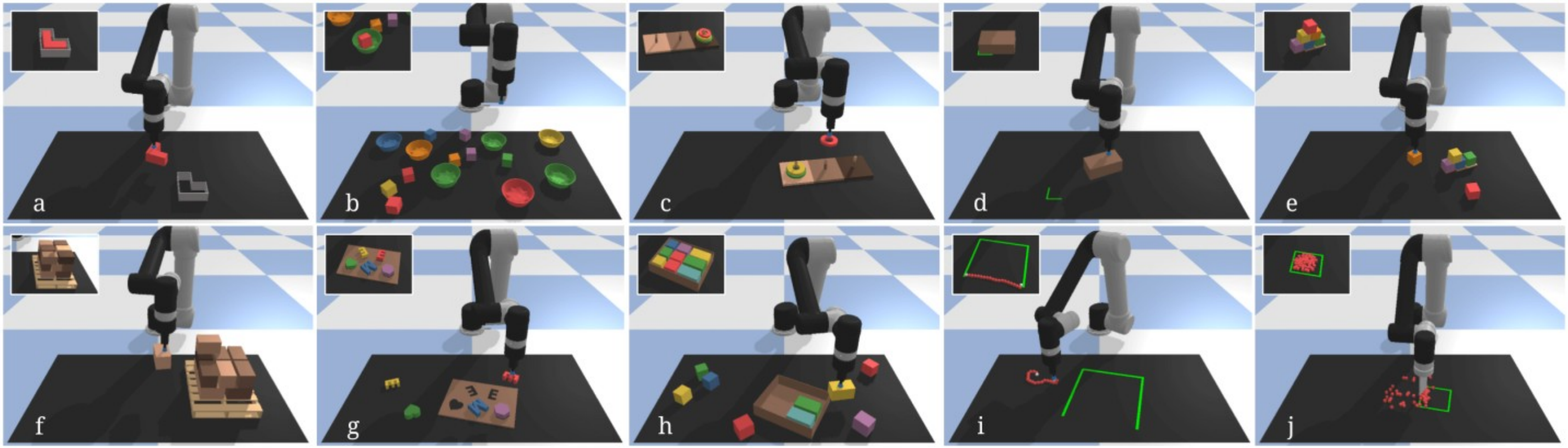
#4) Pick and Place



Equivariant over <u>both</u> pick and place pose: $SE(2) \times SE(2)$

Huang, Wang, Walters, Platt, *Equivariant Transporter Network*, RSS 2022

#4) Pick and Place

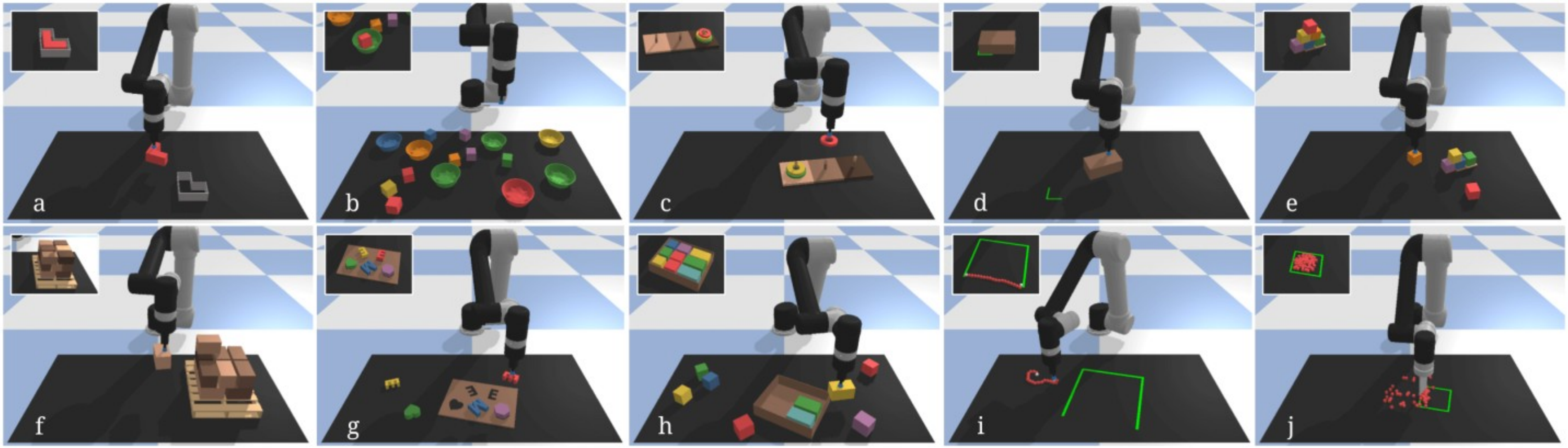


	block-insertion				place-red-in-green				towers-of-hanoi				align-box-corner				stack-block-pyramid			
Method	1	10	100	1000	1	10	100	1000	1	10	100	1000	1	10	100	1000	1	10	100	1000
Equivariant Transporter	**100**	**100**	**100**	**100**	**98.5**	**100**	**100**	**100**	**88.1**	**95.7**	**100**	**100**	41.0	**99.0**	**100**	**100**	**34.6**	**80.0**	**90.8**	**95.1**
Transporter Network	**100**	**100**	**100**	**100**	84.5	**100**	**100**	**100**	73.1	83.9	97.3	98.1	35.0	85.0	97.0	98.0	13.3	42.6	56.2	78.2
Form2Fit	17.0	19.0	23.0	29.0	83.4	**100**	**100**	**100**	3.6	4.4	3.7	7.0	7.0	2.0	5.0	16.0	19.7	17.5	18.5	32.5
Conv. MLP	0.0	5.0	6.0	8.0	0.0	3.0	25.5	31.3	0.0	1.0	1.9	2.1	0.0	2.0	1.0	1.0	0.0	1.8	1.7	1.7
GT-State MLP	4.0	52.0	96.0	99.0	0.0	0.0	3.0	82.2	10.7	10.7	6.1	5.3	47.0	29.0	29.0	59.0	0.0	0.2	1.3	15.3
GT-State MLP 2-Step	6.0	38.0	95.0	**100**	0.0	0.0	19.0	92.8	22.0	6.4	5.6	3.1	**49.0**	12.0	43.0	55.0	0.0	0.8	12.2	17.5

	palletizing-boxes				assembling-kits				packing-boxes				manipulating-rope				sweeping-piles			
	1	10	100	1000	1	10	100	1000	1	10	100	1000	1	10	100	1000	1	10	100	1000
Equivariant Transporter	**75.3**	**98.9**	**99.6**	**99.6**	**63.8**	**90.6**	**98.6**	**100**	**98.3**	**99.4**	**99.6**	**100**	**31.0**	**85.0**	**92.3**	**98.4**	**97.9**	**99.5**	**100**	**100**
Transporter Network	63.2	77.4	91.7	97.9	28.4	78.6	90.4	94.6	56.8	58.3	72.1	81.3	21.9	73.2	85.4	92.1	52.4	74.4	71.5	96.1
Form2Fit	21.6	42.0	52.1	65.3	3.4	7.6	24.2	37.6	29.9	52.5	62.3	66.8	11.9	38.8	36.7	47.7	13.2	15.6	26.7	38.4
Conv. MLP	31.4	37.4	34.6	32.0	0.0	0.2	0.2	0.0	0.3	9.5	12.6	16.1	3.7	6.6	3.8	10.8	28.2	48.4	44.9	45.1
GT-State MLP	0.6	6.4	30.2	30.1	0.0	0.0	1.2	11.8	7.1	1.4	33.6	56.0	5.5	11.5	43.6	47.4	7.2	20.6	63.2	74.4
GT-State MLP 2-Step	0.6	9.6	32.8	37.5	0.0	0.0	1.6	4.4	4.0	3.5	43.4	57.1	6.0	8.2	41.5	58.7	9.7	21.4	66.2	73.9

Huang, Wang, Walters, Platt, *Equivariant Transporter Network*, RSS 2022

#4) Pick and Place

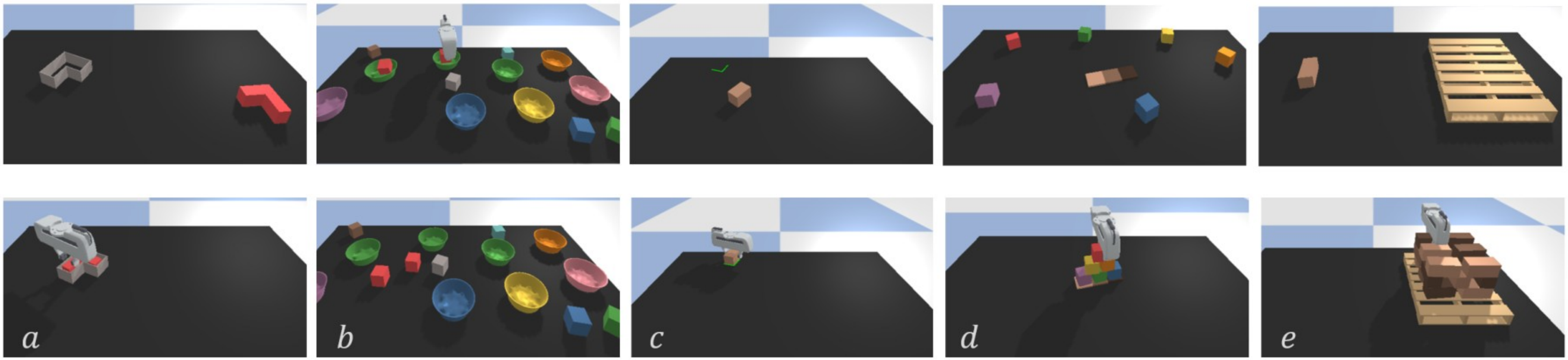


Method	block-insertion 1	10	100	1000	place-red-in-green 1	10	100	1000	towers-of-hanoi 1	10	100	1000	align-box-corner 1	10	100	1000	stack-block-pyramid 1	10	100	1000
Equivariant Transporter	**100**	**100**	**100**	**100**	**98.5**	**100**	**100**	**100**	**88.1**	**95.7**	**100**	**100**	41.0	**99.0**	**100**	**100**	**34.6**	**80.0**	**90.8**	**95.1**
Transporter Network	**100**	**100**	**100**	**100**	84.5	**100**	**100**	**100**	73.1	83.9	97.3	98.1	35.0	85.0	97.0	98.0	13.3	42.6	56.2	78.2
Form2Fit	17.0	19.0	23.0	29.0	83.4	**100**	**100**	**100**	3.6	4.4	3.7	7.0	7.0	2.0	5.0	16.0	19.7	17.5	18.5	32.3
Conv. MLP	0.0	5.0	6.0	8.0	0.0	3.0	25.5	31.3	0.0	1.0	1.9	2.1	0.0	2.0	1.0	1.0	0.0	1.8	1.7	1.7
GT-State MLP	4.0	52.0	96.0	99.0	0.0	0.0	3.0	82.2	10.7	10.7	6.1	5.3	47.0	29.0	29.0	59.0	0.0	0.2	1.3	15.3
GT-State MLP 2-Step	6.0	38.0	95.0	**100**	0.0	0.0	19.0	92.8	22.0	6.4	5.6	3.1	**49.0**	12.0	43.0	55.0	0.0	0.8	12.2	17.5

Method	palletizing-boxes 1	10	100	1000	assembling-kits 1	10	100	1000	packing-boxes 1	10	100	1000	manipulating-rope 1	10	100	1000	sweeping-piles 1	10	100	1000
Equivariant Transporter	**75.3**	**98.9**	**99.6**	**99.6**	**63.8**	**90.6**	**98.6**	**100**	**98.3**	**99.4**	**99.6**	**100**	**31.0**	**85.0**	**92.3**	**98.4**	**97.9**	**99.5**	**100**	**100**
Transporter Network	63.2	77.4	91.7	97.9	28.4	78.6	90.4	94.6	56.8	58.3	72.1	81.3	21.9	73.2	85.4	92.1	52.4	74.4	71.5	96.1
Form2Fit	21.6	42.0	52.1	65.3	3.4	7.6	24.2	37.6	29.9	52.5	62.3	66.8	11.9	38.8	36.7	47.7	13.2	15.6	26.7	38.4
Conv. MLP	31.4	37.4	34.6	32.0	0.0	0.2	0.2	0.0	0.3	9.5	12.6	16.1	3.7	6.6	3.8	10.8	28.2	48.4	44.9	45.1
GT-State MLP	0.6	6.4	30.2	30.1	0.0	0.0	1.2	11.8	7.1	1.4	33.6	56.0	5.5	11.5	43.6	47.4	7.2	20.6	63.2	74.4
GT-State MLP 2-Step	0.6	9.6	32.8	37.5	0.0	0.0	1.6	4.4	4.0	3.5	43.4	57.1	6.0	8.2	41.5	58.7	9.7	21.4	66.2	73.9

Huang, Wang, Walters, Platt, *Equivariant Transporter Network*, RSS 2022

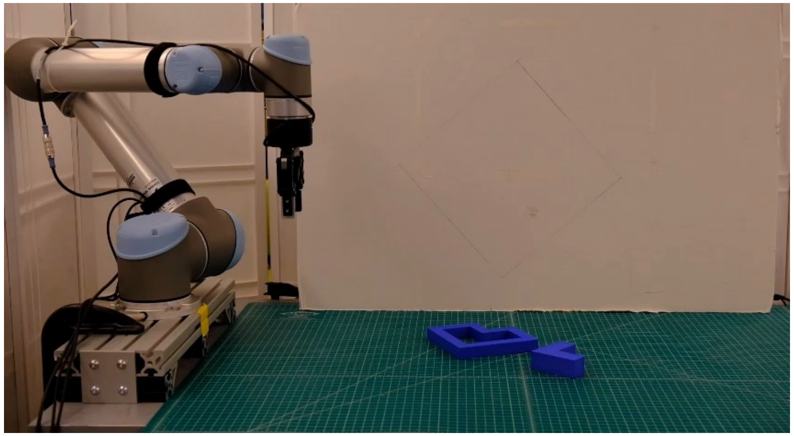
#4) Pick and Place

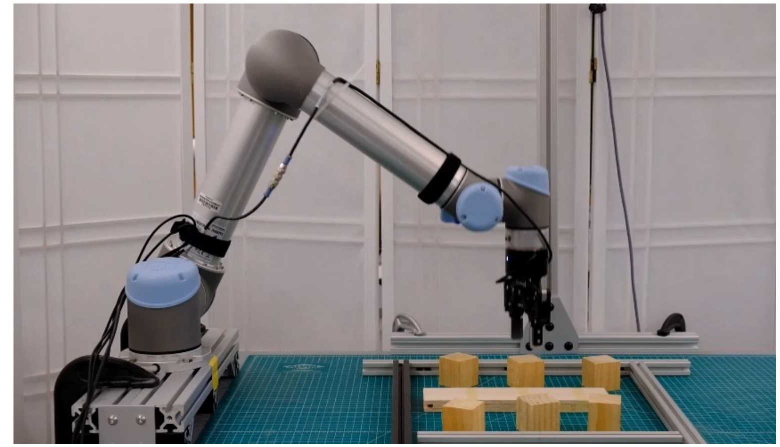


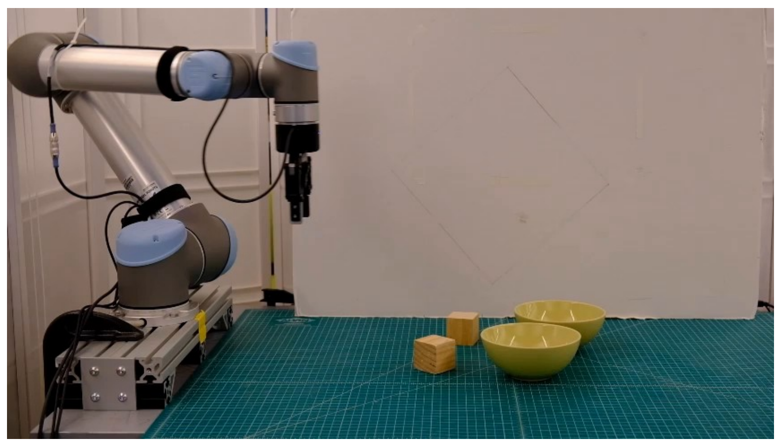
	block-insertion			place-red-in-green			palletizing-boxes			align-box-corner			stack-block-pyramid		
Method	1	10	100	1	10	100	1	10	100	1	10	100	1	10	100
Equivariant Transporter	**100**	**100**	**100**	**95.6**	**100**	**100**	**96.1**	**100**	**100**	**64.0**	**99.0**	**100**	**62.1**	**85.6**	**98.3**
Transporter Network	98.0	**100**	**100**	82.3	94.8	**100**	84.2	99.6	**100**	45.0	85.0	99.0	16.6	63.3	75.0

Huang, Wang, Walters, Platt, *Equivariant Transporter Network*, RSS 2022

#4) Pick and Place



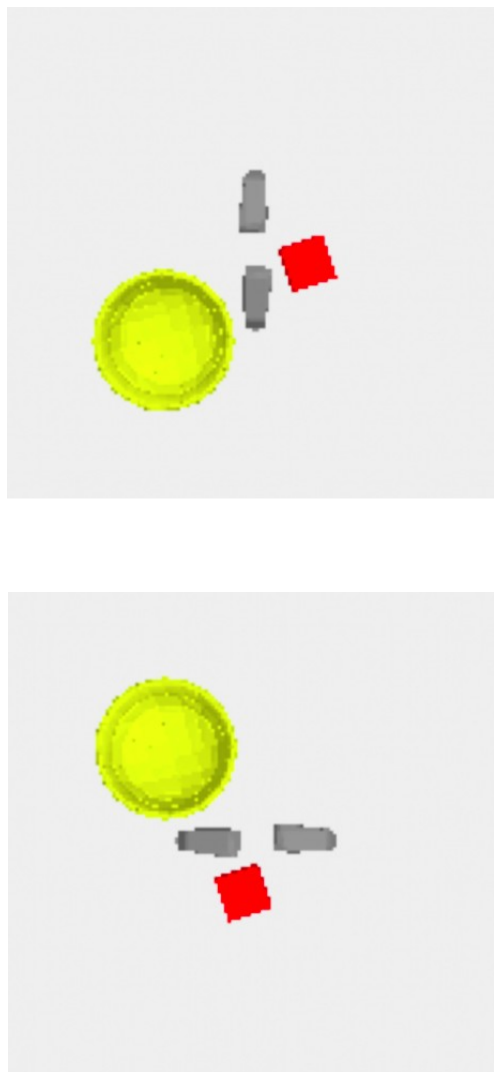




Task	# demos	# completions / # trials	success rate
stack-block-pyramid	10	17/20	95.8%
place-box-in-bowl	10	20/20	100%
block-insertion	10	20/20	100%

Huang, Wang, Walters, Platt, *Equivariant Transporter Network*, RSS 2022

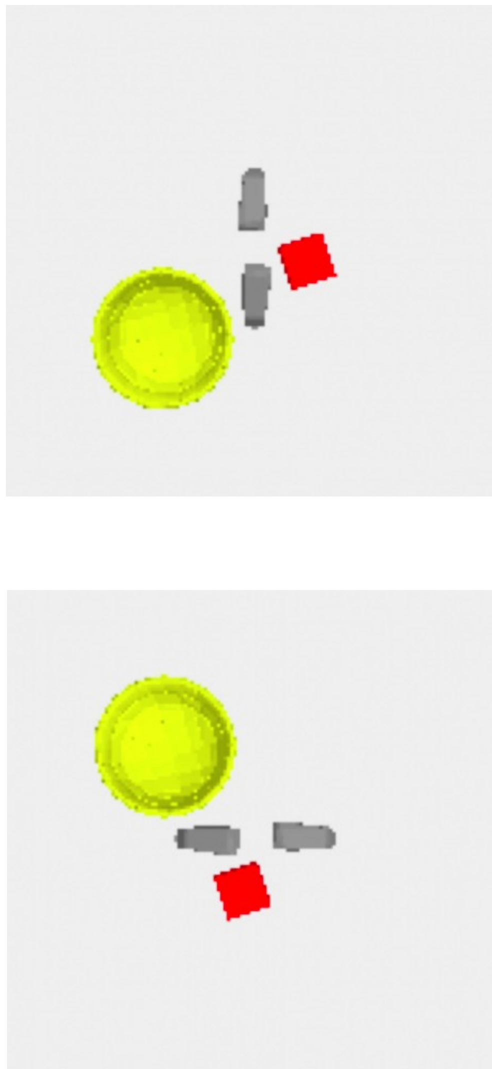
#5) Symmetry Mismatch



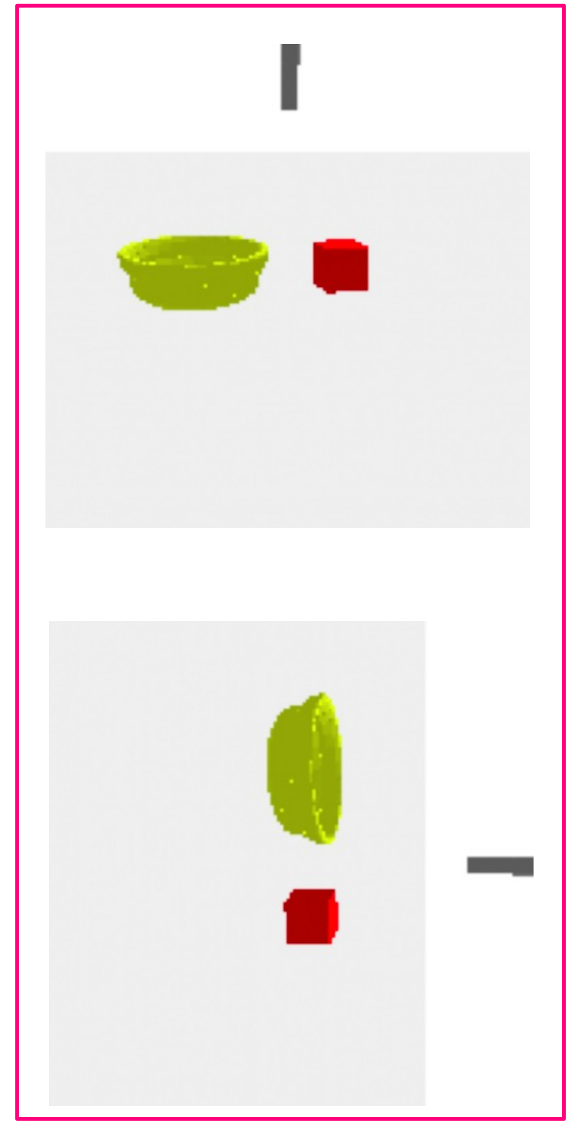
Model symmetry matches domain symmetry

Model symmetry **does not** match domain symmetry

#5) Symmetry Mismatch

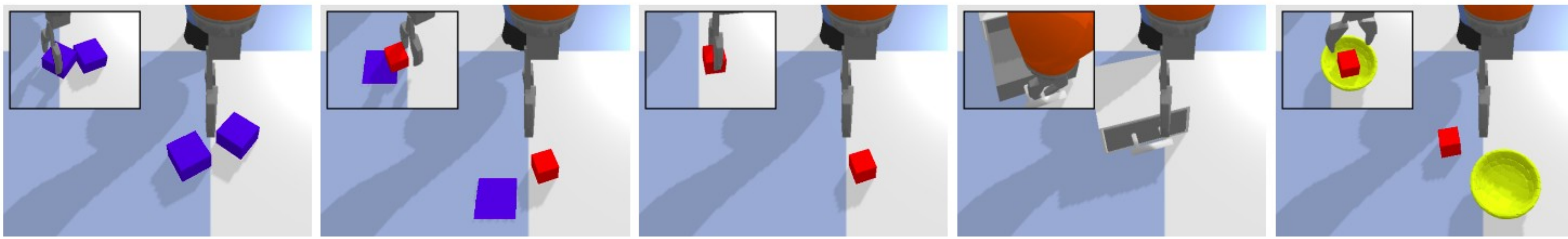


Model symmetry matches domain symmetry

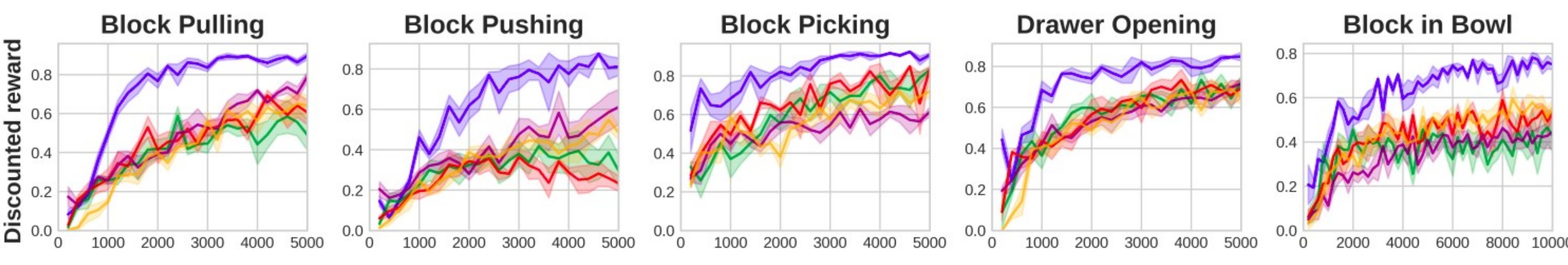


Model symmetry **does not** match domain symmetry

#5) Symmetry Mismatch

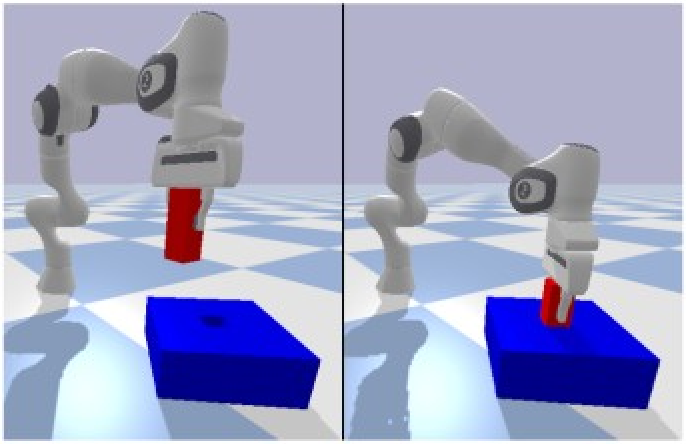


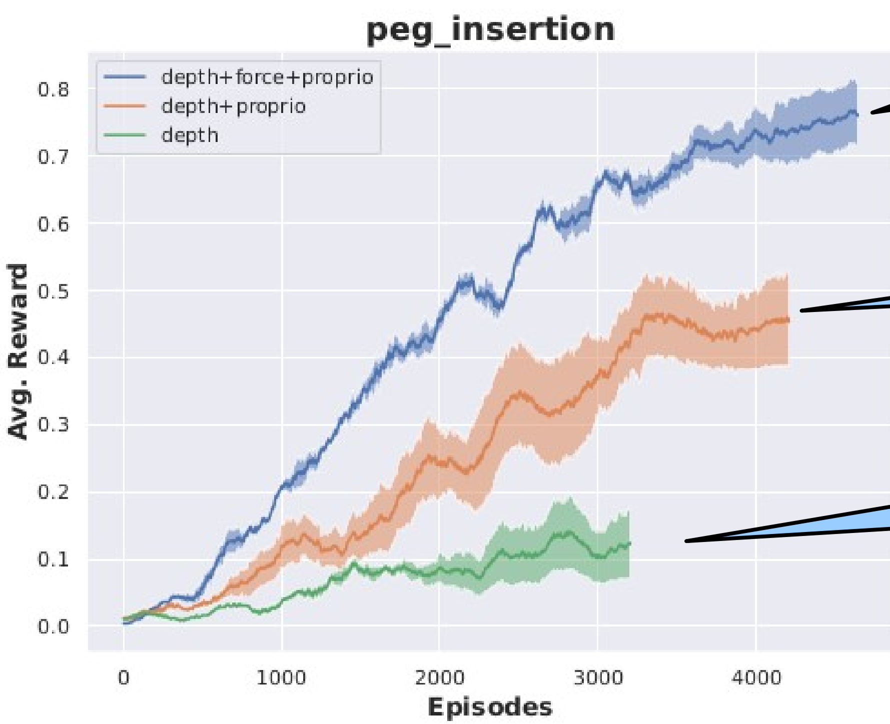
(a) Block Pulling (b) Block Pushing (c) Block Picking (d) Drawer Opening (e) Block in Bowl



Wang, et al., ICLR 2023; Wang, et al., ICML submission

#6) Force Feedback



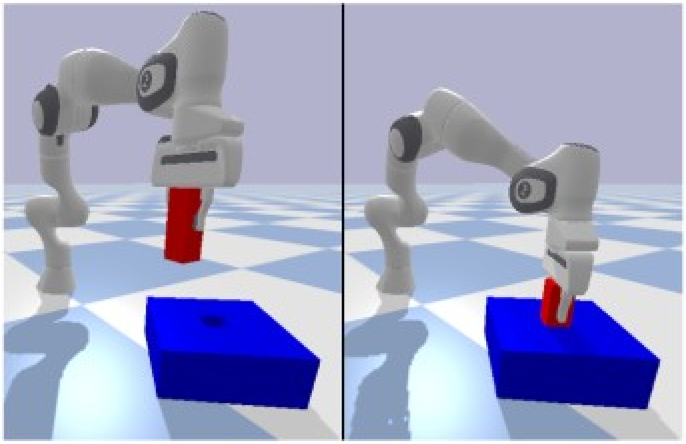


Force + Joint Angles + Vision

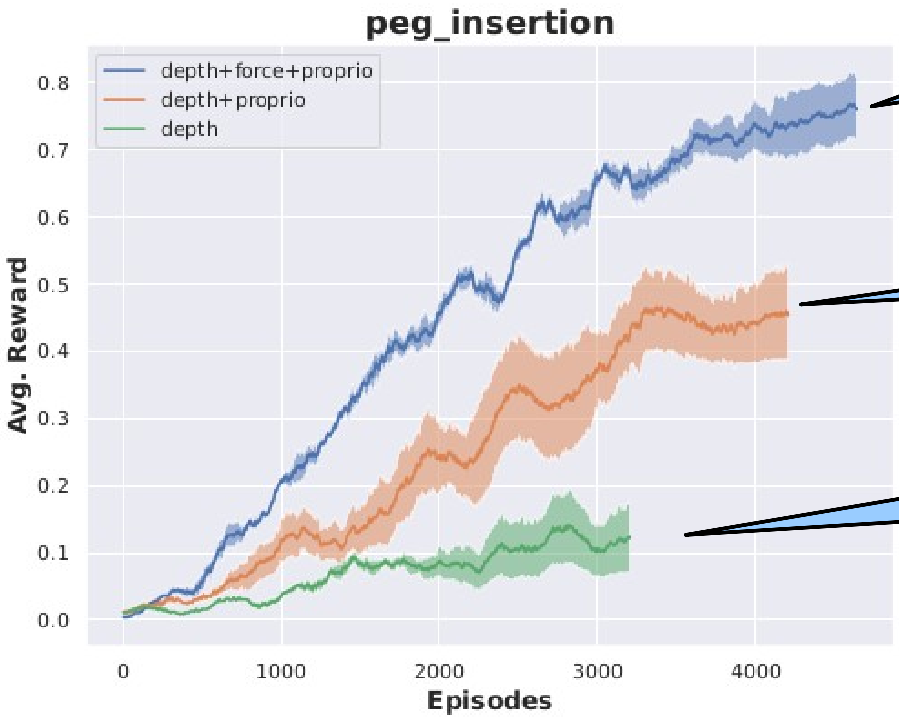
Joint Angles + Vision

Vision

#6) Force Feedback



Imperfect symmetries are essential here!

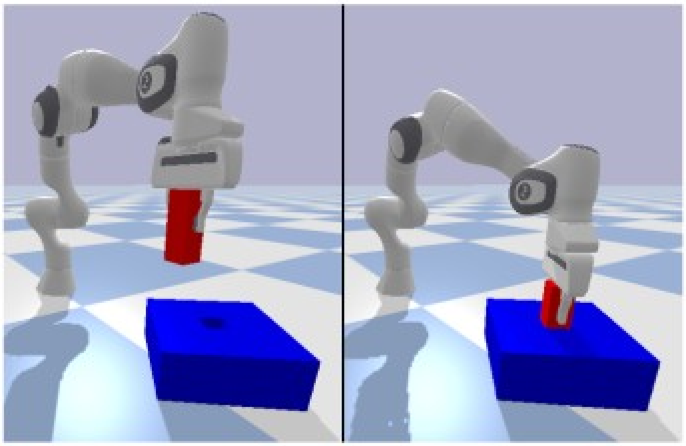


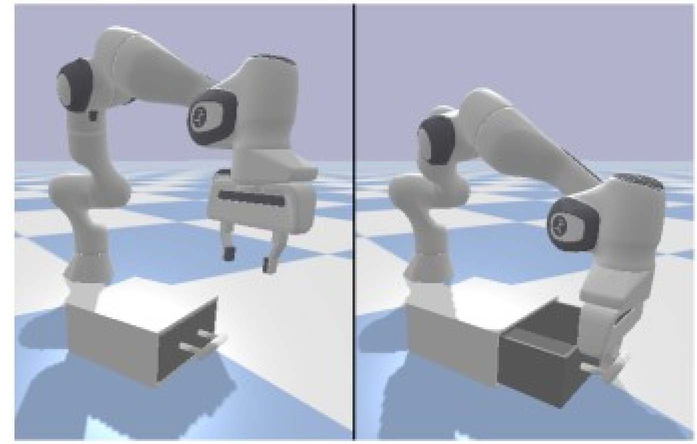
Force + Joint Angles + Vision

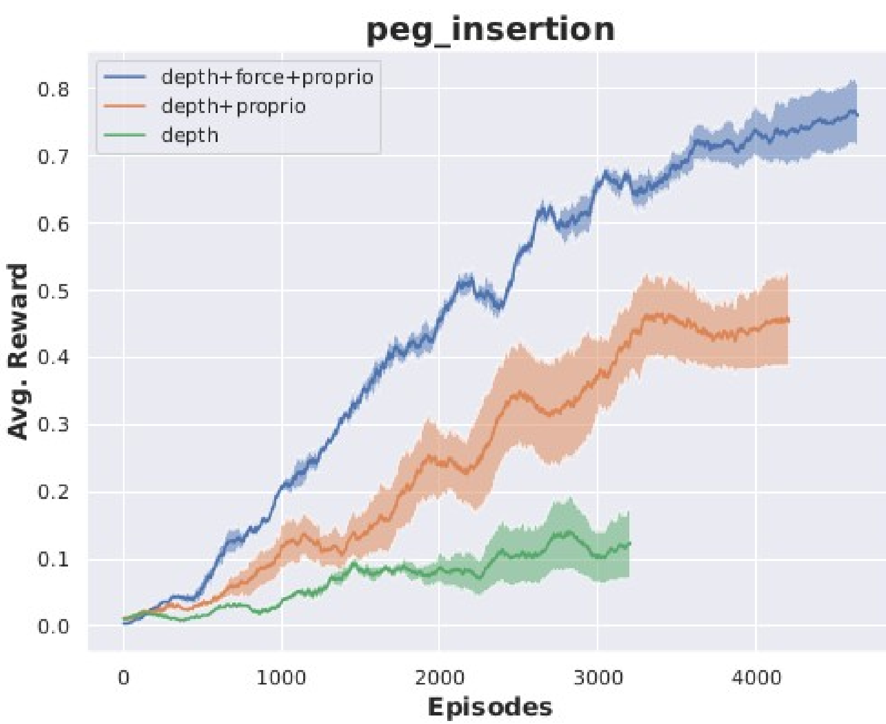
Joint Angles + Vision

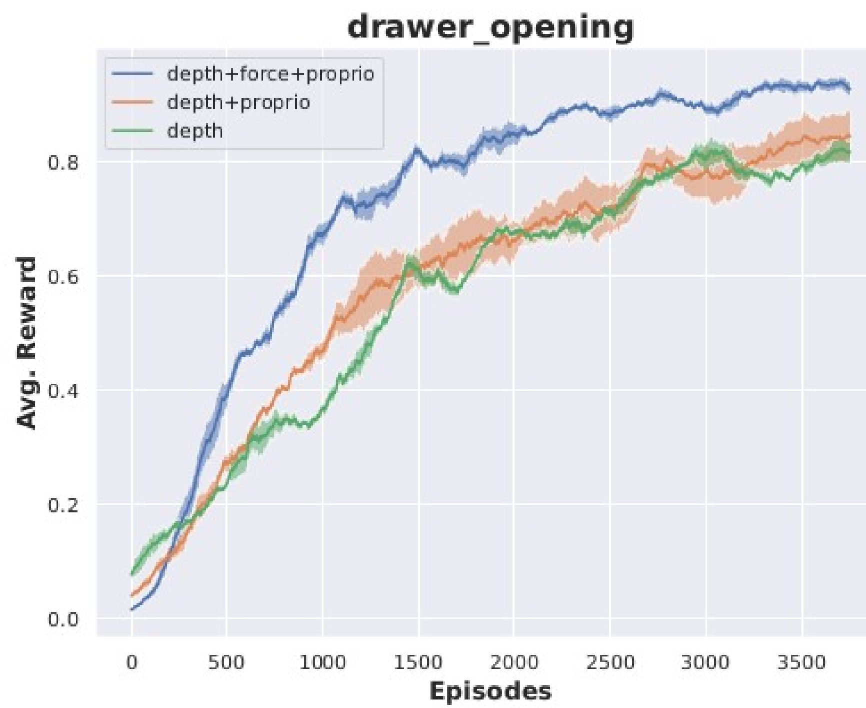
Vision

#6) Force Feedback







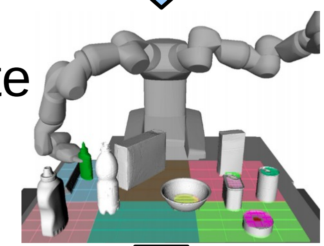


Perspective

Model Based

Detect objects

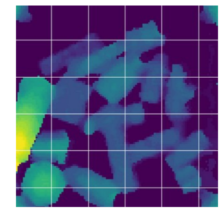
Populate model



Model Based Planner

Powerful, but makes a lot of assumptions

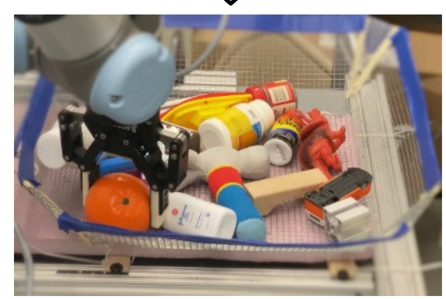
Model Free



Images

Control Policy

Action



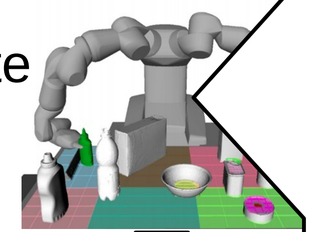
Robust, but hard to scale up

Perspective

Model Based

Detect objects

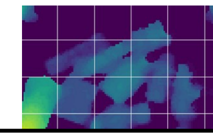
Populate model



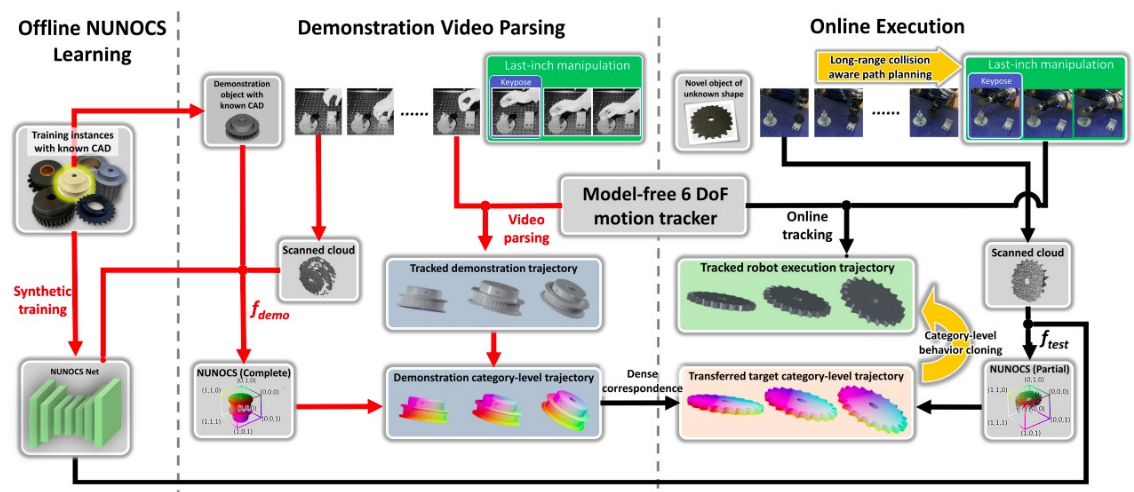
Model Based Planner

Powerful, but makes a lot of assumptions

Model Free



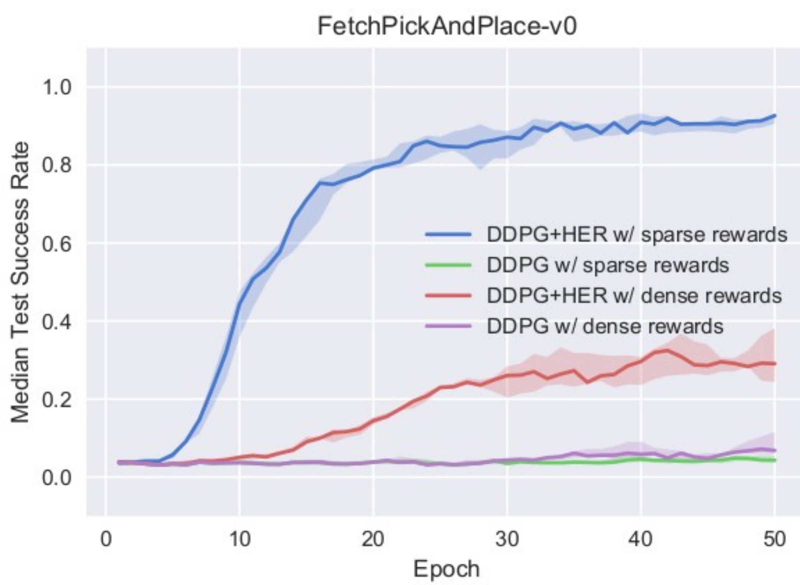
Example: YODO



Wen, Lian, Bekris, Schaal, *You Only Demonstrate Once: Category-Level Manipulation from Single Visual Demonstration,* RSS 2022 Best Paper

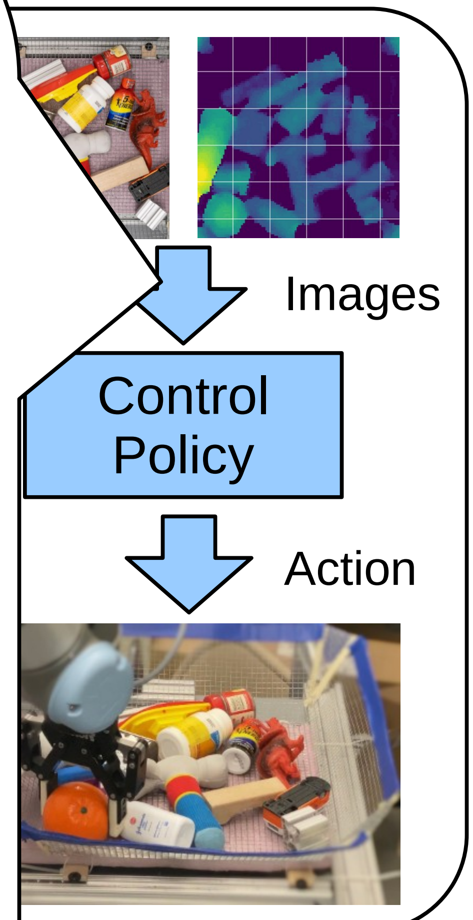
Perspective

Example: OpenAI Baselines



Plappert et al., *Multi-Goal Reinforcement Learning: Challenging Robotics Environments and Request for Research*, ArXiv 2018

Model Free



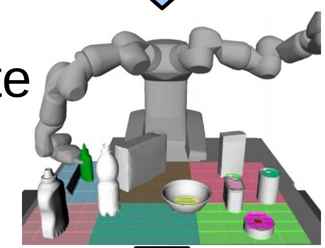
bust, but hard to
cale up

Perspective

Model Based

Detect objects

Populate model



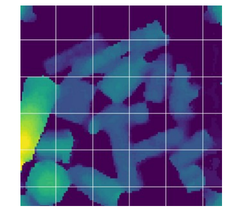
Model Based Planner

A few dozen parameters

?

A few thousand parameters?

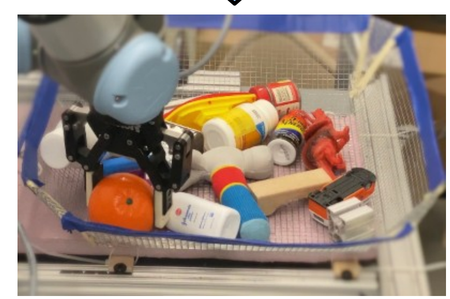
Model Free



Images

Control Policy

Action



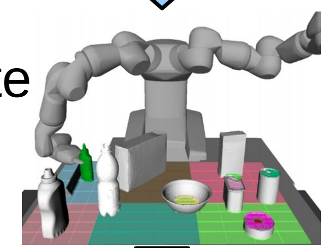
Millions of parameters

Perspective

Model Based

Detect objects

Populate model



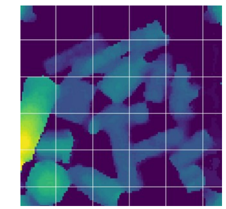
Model Based Planner

A few dozen parameters

Constrained Model Free

Symmetric policies

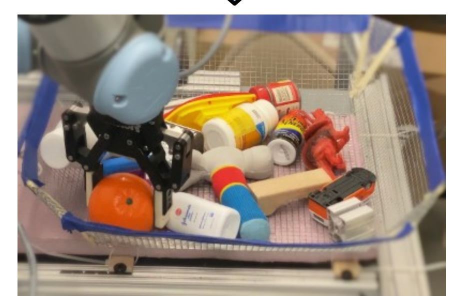
Model Free



Images

Control Policy

Action



Millions of parameters